1. Odvoďte vztahy z následujícího článku:

3.8 Uniform acceleration

The Newtonian definition of a particle moving under uniform acceleration is

$$\frac{du}{dt} = \text{constant}.$$

This turns out to be inappropriate in special relativity since it would imply that $u \to \infty$ as $t \to \infty$, which we know is impossible. We therefore adopt a different definition. Acceleration is said to be **uniform** in special relativity if it has the same value in any **co-moving frame**, that is, at each instant, the acceleration in an inertial frame travelling with the same velocity as the particle has the same value. This is analogous to the idea in Newtonian theory of motion under a constant force. For example, a spaceship whose motor is set at a constant emission rate would be uniformly accelerated in this sense. Taking the velocity of the particle to be $u = u(t)$ relative to an inertial frame S, then at any instant in a co-moving frame S', it follows that $v = u$, the velocity relative to S' is zero, i.e. $u' = 0$, and the acceleration is a constant, a say, i.e. $du'/dt' = a$. Using (3.21), we find

$$\frac{du}{dt} = \frac{1}{\beta^3} a = \left(1 - \frac{u^2}{c^2}\right)^{\frac{3}{2}} a.$$

We can solve this differential equation by separating the variables

$$\frac{du}{(1 - u^2/c^2)^{\frac{3}{2}}} = a\,dt$$

and integrating both sides. Assuming that the particle starts from rest at $t = t_0$, we find

$$\frac{u}{(1 - u^2/c^2)^{\frac{1}{2}}} = a(t - t_0).$$

Solving for u, we get

$$u = \frac{dx}{dt} = \frac{a(t - t_0)}{[1 + a^2(t - t_0)^2/c^2]^{\frac{1}{2}}}.$$

Next, integrating with respect to t, and setting $x = x_0$ at $t = t_0$, produces

$$(x - x_0) = \frac{c}{a}[c^2 + a^2(t - t_0)^2]^{\frac{1}{2}} - \frac{c^2}{a}.$$

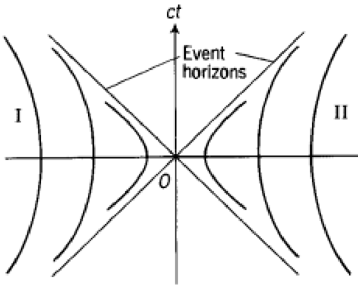

Fig. 3.8 Hyperbolic motions.

This can be rewritten in the form

$$\frac{(x - x_0 + c^2/a)^2}{(c^2/a)^2} - \frac{(ct - ct_0)^2}{(c^2/a)^2} = 1, \tag{3.24}$$

which is a hyperbola in (x, ct)-space. If, in particular, we take $x_0 - c^2/a = t_0 = 0$, then we obtain a family of hyperbolae for different values of a (Fig. 3.8). These world-lines are known as **hyperbolic motions** and, as we shall see in Chapter 23, they have significance in cosmology. It can be shown that the radar distance between the world-lines is a constant. Moreover, consider the regions I and II bounded by the light rays passing through O, and a system of particles undergoing hyperbolic motions as shown in Fig. 3.8 (in some cosmological models, the particles would be galaxies). Then, remembering that light rays emanating from any point in the diagram do so at 45°, no particle in region I can communicate with another particle in region II, and vice versa. The light rays are called **event horizons** and act as barriers beyond which no knowledge can ever be gained. We shall see that event horizons will play an important role later in this book.

2. Spočítejte příklady 3.9,10

3.9 (**§3.8**) A particle moves from rest at the origin of a frame S along the x-axis, with constant acceleration α (as measured in an instantaneous rest frame). Show that the equation of motion is

$$\alpha x^2 + 2c^2 x - \alpha c^2 t^2 = 0,$$

and prove that the light signals emitted after time $t = c/\alpha$ at the origin will never reach the receding particle. A standard clock carried along with the particle is set to read zero at the beginning of the motion and reads τ at time t in S. Using the clock hypothesis, prove the following relationships:

$$\frac{u}{c} = \tanh\frac{\alpha\tau}{c}, \qquad \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} = \cosh\frac{\alpha\tau}{c},$$

$$\frac{\alpha t}{c} = \sinh\frac{\alpha\tau}{c}, \qquad x = \frac{c^2}{\alpha}\left(\cosh\frac{\alpha\tau}{c} - 1\right).$$

Show that, if $T^2 \ll c^2/\alpha^2$, then, during an elapsed time T in the inertial system, the particle clock will record approximately the time $T(1 - \alpha^2 T^2/6c^2)$.

If $\alpha = 3g$, find the difference in recorded times by the spaceship clock and those of the inertial system

(a) after 1 hour;
(b) after 10 days.

3.10 (**§3.9**) A space traveller $\bar{A}$ travels through space with uniform acceleration g (to ensure maximum comfort). Find the distance covered in 22 years of $\bar{A}$'s time. [Hint: using years and light years as time and distance units, respectively, then $g = 1.03$]. If on the other hand, $\bar{A}$ describes a straight double path $XYZYX$, with acceleration g on XY and ZY, and deceleration on YZ and YX, for 6 years each, then draw a space-time diagram as seen from the Earth and find by how much the Earth would have aged in 24 years of $\bar{A}$'s time.