Lambert's cosine law

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In optics, **Lambert's cosine law** says that the radiant intensity or luminous intensity observed from an ideal diffusely reflecting surface or ideal diffuse radiator is directly proportional to the cosine of the angle θ between the direction of the incident light and the surface normal.[1][2] The law is also known as the **cosine emission law**[3] or **Lambert's emission law**. It is named after Johann Heinrich Lambert, from his *Photometria*, published in 1760.[4]

A surface which obeys Lambert's law is said to be *Lambertian*, and exhibits Lambertian reflectance. Such a surface has the same radiance when viewed from any angle. This means, for example, that to the human eye it has the same apparent brightness (or luminance). It has the same radiance because, although the emitted power from a given area element is reduced by the cosine of the emission angle, the apparent size (solid angle) of the observed area, as seen by a viewer, is decreased by a corresponding amount. Therefore, its radiance (power per unit solid angle per unit projected source area) is the same.

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When an area element is radiating as a result of being illuminated by an external source, the irradiance (energy or photons/time/area) landing on that area element will be proportional to the cosine of the angle between the illuminating source and the normal. A Lambertian scatterer will then scatter this light according to the same cosine law as a Lambertian emitter. This means that although the radiance of the surface depends on the angle from the normal to the illuminating source, it will not depend on the angle from the normal to the observer. For example, if the moon were a Lambertian scatterer, one would expect to see its scattered brightness appreciably diminish towards the terminator due to the increased angle at which sunlight hit the surface. The fact that it does not diminish illustrates that the moon is not a Lambertian scatterer, and in fact tends to scatter more light into the oblique angles than would a Lambertian scatterer.

The emission of a Lambertian radiator does not depend upon the amount of incident radiation, but rather from radiation originating in the emitting body itself. For example, if the sun were a Lambertian radiator, one would expect to see a constant brightness across the entire solar disc. The fact that the sun exhibits limb darkening in the visible region illustrates that it is not a Lambertian radiator. A black body is an example of a Lambertian radiator.

Details of equal brightness effect

The situation for a Lambertian surface (emitting or scattering) is illustrated in Figures 1 and 2. For conceptual clarity we will think in terms of photons rather than energy or luminous energy. The wedges in the circle each represent an equal angle $d\Omega$, and for a Lambertian surface, the number of photons per second emitted into each wedge is proportional to the area of the wedge.

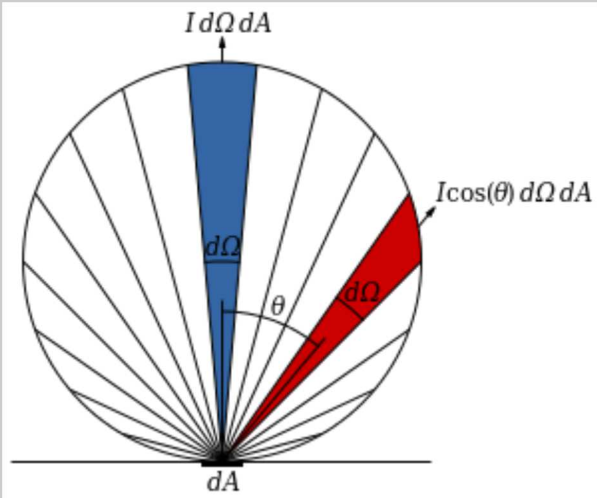

Figure 1: Emission rate (photons/s) in a normal and off-normal direction. The number of photons/sec directed into any wedge is proportional to the area of the wedge.

It can be seen that the length of each wedge is the product of the diameter of the circle and $\cos(\theta)$. It can also be seen that the maximum rate of photon emission per unit solid angle is along the normal and diminishes to zero for $\theta = 90°$. In mathematical terms, the radiance along the normal is I photons/(s·cm^2·sr) and the number of photons per second emitted into the vertical wedge is $I\,d\Omega\,dA$. The number of photons per second emitted into the wedge at angle θ is $I\cos(\theta)\,d\Omega\,dA$.

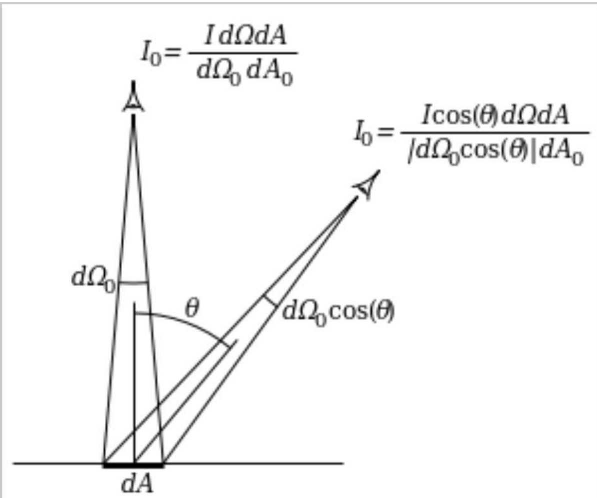

Figure 2: Observed intensity (photons/(s·cm^2·sr)) for a normal and off-normal observer; dA_0 is the area of the observing aperture and $d\Omega$ is the solid angle subtended by the aperture from the viewpoint of the emitting area element.

Figure 2 represents what an observer sees. The observer directly above the area element will be seeing the scene through an aperture of area dA_0 and the area element dA will subtend a (solid) angle of $d\Omega_0$. We can assume without loss of generality that the aperture happens to subtend solid angle $d\Omega$ when "viewed" from the emitting area element. This normal observer will then be recording $I\,d\Omega\,dA$ photons per second and so will be measuring a radiance of

$$I_0 = \frac{I\,d\Omega\,dA}{d\Omega_0\,dA_0} \text{ photons/(s}\cdot\text{cm}^2\cdot\text{sr).}$$

The observer at angle θ to the normal will be seeing the scene through the same aperture of area dA_0 and the area element dA will subtend a (solid) angle of $d\Omega_0\cos(\theta)$. This observer will be recording $I\cos(\theta)\,d\Omega\,dA$ photons per second, and so will be measuring a radiance of

$$I_0 = \frac{I\cos(\theta)\,d\Omega\,dA}{d\Omega_0\,\cos(\theta)\,dA_0} = \frac{I\,d\Omega\,dA}{d\Omega_0\,dA_0}$$

photons/(s·cm^2·sr),

which is the same as the normal observer.

Relating peak luminous intensity and luminous flux

In general, the luminous intensity of a point on a surface varies by direction; for a Lambertian surface, that distribution is defined by the cosine law, with peak luminous intensity in the normal direction. Thus when the Lambertian assumption holds, we can calculate the total luminous flux, F_{tot}, from the peak luminous intensity, I_{max}, by integrating the cosine law:

$$F_{tot} = \int_0^{\pi/2} \int_0^{2\pi} \cos(\theta) I_{max}\,\sin(\theta)\,\mathrm{d}\phi\,\mathrm{d}\theta$$

$$= 2\pi \cdot I_{max} \int_0^{\pi/2} \cos(\theta)\sin(\theta)\ \mathrm{d}\,\theta$$

$$= 2\pi \cdot I_{max} \int_0^{\pi/2} \frac{\sin(2\theta)}{2}\ \mathrm{d}\,\theta$$

and so

$$F_{tot} = \pi\,\mathrm{sr} \cdot I_{max}$$

where $\sin(\theta)$ is the determinant of the Jacobian matrix for the unit sphere, and realizing that I_{max} is luminous flux per steradian.[5] Similarly, the peak intensity will be $1/(\pi\,\mathrm{sr})$ of the total radiated luminous flux. For Lambertian surfaces, the same factor of $\pi\,\mathrm{sr}$ relates luminance to luminous emittance, radiant intensity to radiant flux, and radiance to radiant emittance. Radians and steradians are, of course, dimensionless and so "rad" and "sr" are included only for clarity.

Example: A surface with a luminance of say 100 cd/m^2 (= 100 nits, typical PC monitor) will, if it is a perfect Lambert emitter, have a luminous emittance of 314 lm/m^2. If its area is 0.1 m^2 (~19" monitor) then the total light emitted, or luminous flux, would thus be 31.4 lm.

Uses

Lambert's cosine law in its reversed form (Lambertian reflection) implies that the apparent brightness of a Lambertian surface is proportional to the cosine of the angle between the surface normal and the direction of the incident light.

This phenomenon is, among others, used when creating mouldings, with the effect of creating light- and dark-shaded stripes on a structure or object without having to change the material or apply pigment. The contrast of dark and light areas gives definition to the object. Mouldings are strips of material with various cross-sections used to cover transitions between surfaces or for decoration.

See also

- Transmittance
- Reflectivity
- Passive solar building design
- Sun path

References

1. RCA Electro-Optics Handbook, p.18 ff
2. Modern Optical Engineering, Warren J. Smith, McGraw-Hill, p.228, 256
3. Pedrotti & Pedrotti (1993). *Introduction to Optics*. Prentice Hall. ISBN 0135015456.
4. Lambert, J H (1760). *Photometria, sive de Mensura et gradibus luminis, colorum et umbrae*. (https://archive.org/details/lambertsphotome00lambgoog)
5. Incropera and DeWitt, *Fundamentals of Heat and Mass Transfer*, 5th ed., p.710.

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