Funkce dvou proměnných

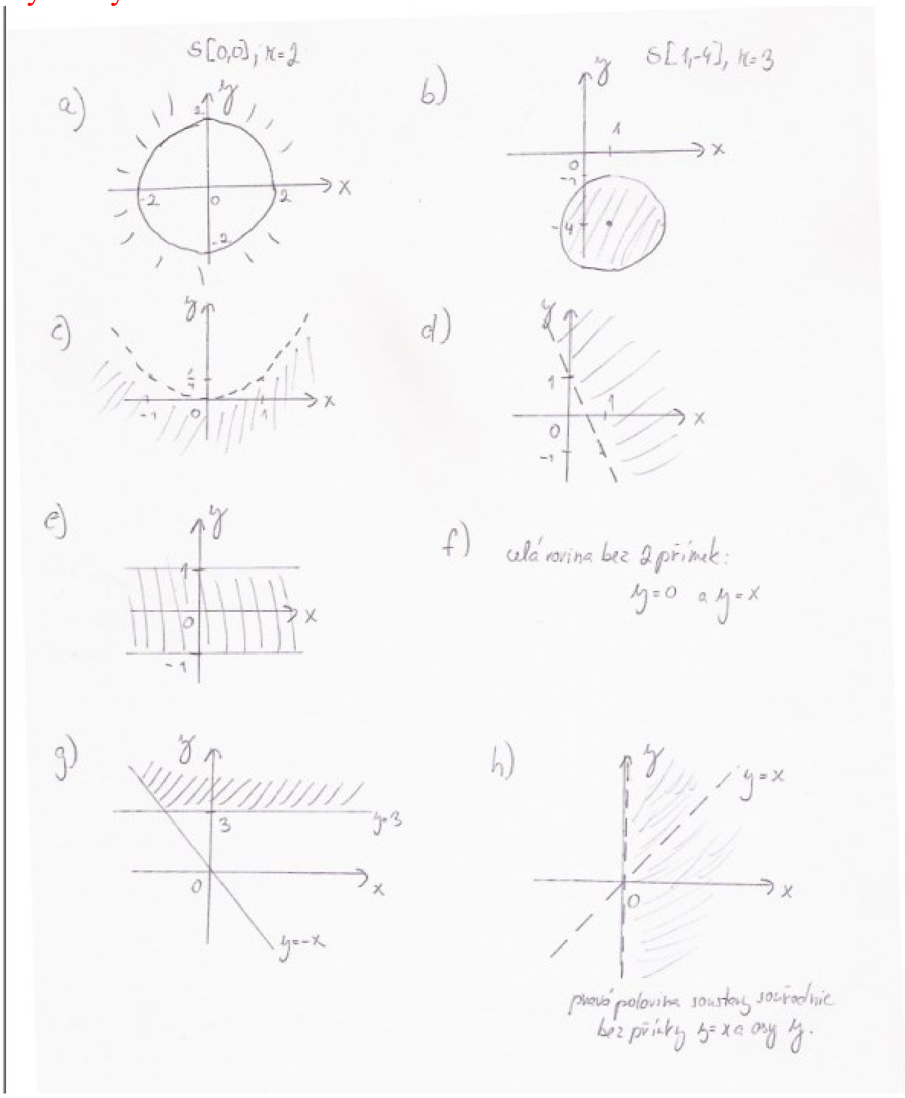
1. Určete definiční obor funkcí dvou proměnných:
   a) $f(x,y)=\sqrt{x^2+y^2-4}$
   b) $f(x,y)=\sqrt{-x^2+2x-y^2-8y-8}$
   c) $f(x,y)=\ln(x^2-4y)$
   d) $f(x,y)=\ln(2x+y-1)$
   e) $f(x,y)=x+\arccos y$
   f) $f(x,y)=\frac{5}{x-y}+\frac{x}{y}$
   g) $f(x,y)=\sqrt{x+y}+\sqrt{y-3}$
   h) $f(x,y)=\frac{\ln(xy^2)}{x-y}$

2. Určete parciální derivace funkcí:
   a) $f(x,y)=x^2+2y^2$
   b) $f(x,y)=yx^2+\cos y$
   c) $f(x,y)=\sqrt{x^2+y^2+5}$
   d) $f(x,y)=\ln(xy+y^4)$
   e) $f(x,y)=x\ln(y+x)$
   f) $f(x,y)=\sin(xy)$

3. Vypočtěte parciální derivace prvních a druhých řádů funkce
   a) $f(x,y)=x^2+y^2+1$
   b) $f(x,y)=x^3+2x^2y^2+x$
   c) $f(x,y)=\ln xy$
   d) $f(x,y)=arctg\,\frac{x}{y}$

4. Vypočtěte parciální derivace prvního a druhého řádu v bodě C:
   a) $f(x,y)=x^2+5y^2+x$, C [1,2]
   b) $f(x,y)=x^3y^2+y^2$, C [-2,3]

Výsledky:



2:

a) $\frac{\partial f}{\partial x} = 2x$, $\frac{\partial f}{\partial y} = 4y$ b) $\frac{\partial f}{\partial x} = 2xy$, $\frac{\partial f}{\partial y} = x^2 - \sin y$, c) $\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2+5}}$, $\frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2+5}}$, d) $\frac{\partial f}{\partial x} = \frac{y}{xy+y^4} = \frac{1}{x+y^3}$, $\frac{\partial f}{\partial y} = \frac{x+4y^3}{xy+y^4}$, e) $\frac{\partial f}{\partial x} = \ln(x+y) + \frac{x}{x+y}$, $\frac{\partial f}{\partial y} = \frac{x}{x+y}$, f) $\frac{\partial f}{\partial x} = y\cos xy$, $\frac{\partial f}{\partial y} = x\cos xy$

3:

a) $\frac{\partial f}{\partial x} = 2x$, $\frac{\partial f}{\partial y} = 2y$, $\frac{\partial^2 f}{\partial x^2} = 2$, $\frac{\partial^2 f}{\partial y^2} = 2$, $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 0$

b) $\frac{\partial f}{\partial x} = 3x^2 + 4xy^2 + 1$, $\frac{\partial f}{\partial y} = 4x^2 y$, $\frac{\partial^2 f}{\partial x^2} = 6x + 4y^2$, $\frac{\partial^2 f}{\partial y^2} = 8x$, $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 8xy$

c) $\frac{\partial f}{\partial x} = \frac{1}{x}$, $\frac{\partial f}{\partial y} = \frac{1}{y}$, $\frac{\partial^2 f}{\partial x^2} = -\frac{1}{x^2}$, $\frac{\partial^2 f}{\partial y^2} = -\frac{1}{y^2}$, $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 0$

d) $\frac{\partial f}{\partial x}=\frac{y}{x^2+y^2}$, $\frac{\partial f}{\partial y}=-\frac{x}{x^2+y^2}$, $\frac{\partial^2 f}{\partial x^2}=-\frac{2xy}{\left(x^2+y^2\right)^2}$, $\frac{\partial^2 f}{\partial y^2}=\frac{2xy}{\left(x^2+y^2\right)^2}$,

$\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}=\frac{x^2-y^2}{\left(x^2+y^2\right)^2}$

4: a) $\frac{\partial f}{\partial x}(1,2)=3$, $\frac{\partial f}{\partial y}(1,2)=20$, $\frac{\partial^2 f}{\partial x^2}(1,2)=2$, $\frac{\partial^2 f}{\partial y^2}(1,2)=10$, $\frac{\partial^2 f}{\partial x\partial y}(1,2)=\frac{\partial^2 f}{\partial y\partial x}(1,2)=0$

b) $\frac{\partial f}{\partial x}(-2,3)=108$, $\frac{\partial f}{\partial y}(-2,3)=-42$, $\frac{\partial^2 f}{\partial x^2}(-2,3)=-108$, $\frac{\partial^2 f}{\partial y^2}(-2,3)=-14$,

$\frac{\partial^2 f}{\partial x\partial y}(-2,3)=\frac{\partial^2 f}{\partial y\partial x}(-2,3)=72$