SILESIAN UNIVERSITY
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Mathematics in economics

Lecture 9

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Geometric series and its application

Geometric series belong among the most simple, yet the most used series in practice.

Definition: $$\frac{a_{n+}}{a_n} = \forall \in$$

Simply put, elements of a geometric series is generated by multiplying each element by a constant q called the *quotient*.

Geometric series – numerical examples

Examples of geometric series:

$$2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \ldots$$

In this series $a_1 = 2$ and $q = 2/3$.

Another example:

$$1, 2, 4, 8, 16, \ldots$$

Here, $a_1 = 1$ and $q = 2$.

Geometric series and its sum

The sum of first *n* numbers of a geometric series is given as:

$$S_n =$$

The sum of a convergent infinite geometric series is given as:

$$S =$$

Note: The latter formula is obtained as a limit to infinity of the former formula.

Geometric series and its convergence

Geometric series is convergent if and only if:

$$|q| < 1$$

Only in this case it is guaranteed elements of a series are approaching 0 as n approaches infinity.

Also, the formula for the sum can be used only for convergent series!

Geometric series – Problem 1

Decide convergence and find the sum of the series:

$$S = + + + \quad +$$

We see that $a_1 = 1$ and $q = 1/3$. The series is convergent.

Its sum:

$$S = \frac{a}{-} \quad \frac{1}{-} \quad \frac{3}{-}$$

Geometric series – Problem 2

Decide convergence and find the sum of the series:

$$\sum$$

We see that $a_1 = 2/5$ and also $q = 2/5$. The series is convergent.

Its sum:

$$s = \frac{\frac{2}{5}}{1 - \ldots} = \frac{2}{\ldots}$$

Geometric series – Problem 3

Decide convergence and find the sum of the series:

$$\sum$$

We see that a_1 = 4/3 and also q = 4/3. The series is divergent! Its sum is infinity.

We cannot use the formula for its sum! It is a common mistake in tests!

Geometric series – Problem 4

Decide convergence and find the sum of the series:

$$\sum$$

We see that a_1 = 1 (!) and q = 3/4. The series is convergent.

Its sum:

$$S =$$

Geometric series and its application in economics

A total revenue of a book or a movie: when a new book/movie is released, the highest revenue comes from the first week, and then the revenue decreases usually by 30-40%.

This situation can be modelled by an infinte geometric series, which gives a very good approximation of real revenues.

Geometric series and its application in economics

Example: Find the total revenue of a movie which earns 70 mil dollar during the first week, while earnings decrease by 30 % each week.

Solution: We have a geometric series

$$S = \quad + \quad + \quad\quad + \quad +$$

Clearly, $a_1 = 70$ and $q = 0.7$.

$$S = \frac{a}{-} \quad \frac{}{-}$$

Geometric series and its application in economics

Example: Find the total revenue of a book which earns 0,5 mil dollar during the first week, while earnings decrease by 20 % each week.

Solution: We have a geometric series

$$0.5 + \quad + \quad + \quad +$$

Clearly, $a_1 = 0.5$ and $q = 0.8$.

$$S = \frac{0.5}{\quad}$$

Geometric series and its application

Other applications of geometric series include present values of annuities, bonds, etc.

Generalization of number series is function series, which will be dealt in the second part of this lecture and Lecture 10.

Geometric series – Problems to solve 1

Is the series a geometric series? If yes, find its sum:

$\sum$

$\sum$

$\sum$

$\sum$

Geometric series – Problems to solve 2

Is the series a geometric series? If yes, find its sum:

$\sum$ $\sum$

$\sum$

$\sum$

$\sum$

Geometric series – Problems to solve 3

A movie earns 45 million dollars on its opening week. Find the total revenue of the movie for a decrease of:

a) 20%
b) 30%
c) 40%
d) 50% weekly.

Function series – an introduction

In the second part of this lecture we will introduce function series, which are a generalization of number series.

By a function series we mean the following series:

$$\sum_{n=1}^{\infty} f_n(x) = f_1(x) + f_2(x) + \cdots$$

Function series – an introduction

The main goal is to find x for which a function series converges. This set is called range of convergence.

Example: Let $\sum \quad + \quad + \quad +$ be a given series.

Now, let x = ½. Then, we obtain: $\sum \quad 1 \quad 1 \quad 1 \quad + + +$

This series is convergent.

However, when x = 2, we get: $\sum \quad + \quad +$

which is divergent.

Function series
Power series

The simplest function series are power series:

$$\sum \quad = + \quad - \quad + \quad - \quad +$$

Power series converge on an interval $a -\quad +$, where *a* is called a centre of a series and ρ is a radius of a convergence.

Function series
Power series – cont.

The radius of convergence is given by the following formulas:

$$\rho = \text{[illegible]}$$

$$\rho = \text{[illegible]}$$

A convergence of a series in points $a + \rho$ and $a - \rho$ must be resolved separately.

Function series
Power series – Problem 1

Find the interval of convergence of the series $\sum$.

Solution:

We see that a = 2 and $c_n =$.

$$\rho = \quad = \quad =$$

Hence: $IK = \; - \; + \; = \; - \; + \; =$

It can be shown the series is divergent at -1 and +1.

Function series
Power series – Problem 2

Find the interval of convergence of the series $\sum$.

Solution:
We see that a = -5 and $c_n =$.

$$\rho = \quad = \quad + \quad +$$

Therefore, the series is convergent only at its center (-5).

Function series
Power series – Problem 3

Find the interval of convergence of the series $\sum$.

Solution:
We see that a = -1 and $c_n =$.

$$\rho = \quad = \quad 1 \quad = \quad 1 \quad \sqrt[n]{n^3} \quad =$$

Hence: $IK = \quad = - \quad + \quad = - - - + = -$

Function series
Power series – Problem to solve

Find the interval of convergence of the series:

$$\sum_{n=1}^{\infty} n x^n$$

Thank you for your attention