Growing annuity & Perpetuity

Lecture for Corporate Finance

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SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

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Outline of the lecture

- PV and FV of growing annuity
- PV of perpetuity
- Selected interesting combinations

Growing annuity

The present value of a growing annuity formula calculates the present day value of a series of future periodic payments that grow at a proportionate rate. A growing annuity may sometimes be referred to as an increasing annuity.

For a growing annuity, each cash flow increases at a certain rate. This formula is the general formula for summing the discounted future cash flows along with using **1 + g** to factor in that each future cash flow will increase at a specific rate.

Like all financial formulas that involve a rate, it is important to correlate the rate per period to the number of periods in the present value of a growing annuity formula. If the payments are monthly, then the rate would need to be the monthly rate (=EAIR).

The future value of a growing annuity can be calculated by working out each individual cash flow by (a) growing the initial cash flow at **g**; (b) finding future value of each cash flow at the interest rate **i** and (c) then summing up all the component future values. The future value of a growing annuity can also be calculated by growing the present value of the growing annuity at **i** for **n** periods.

Growing vs. Ordinary Annuity

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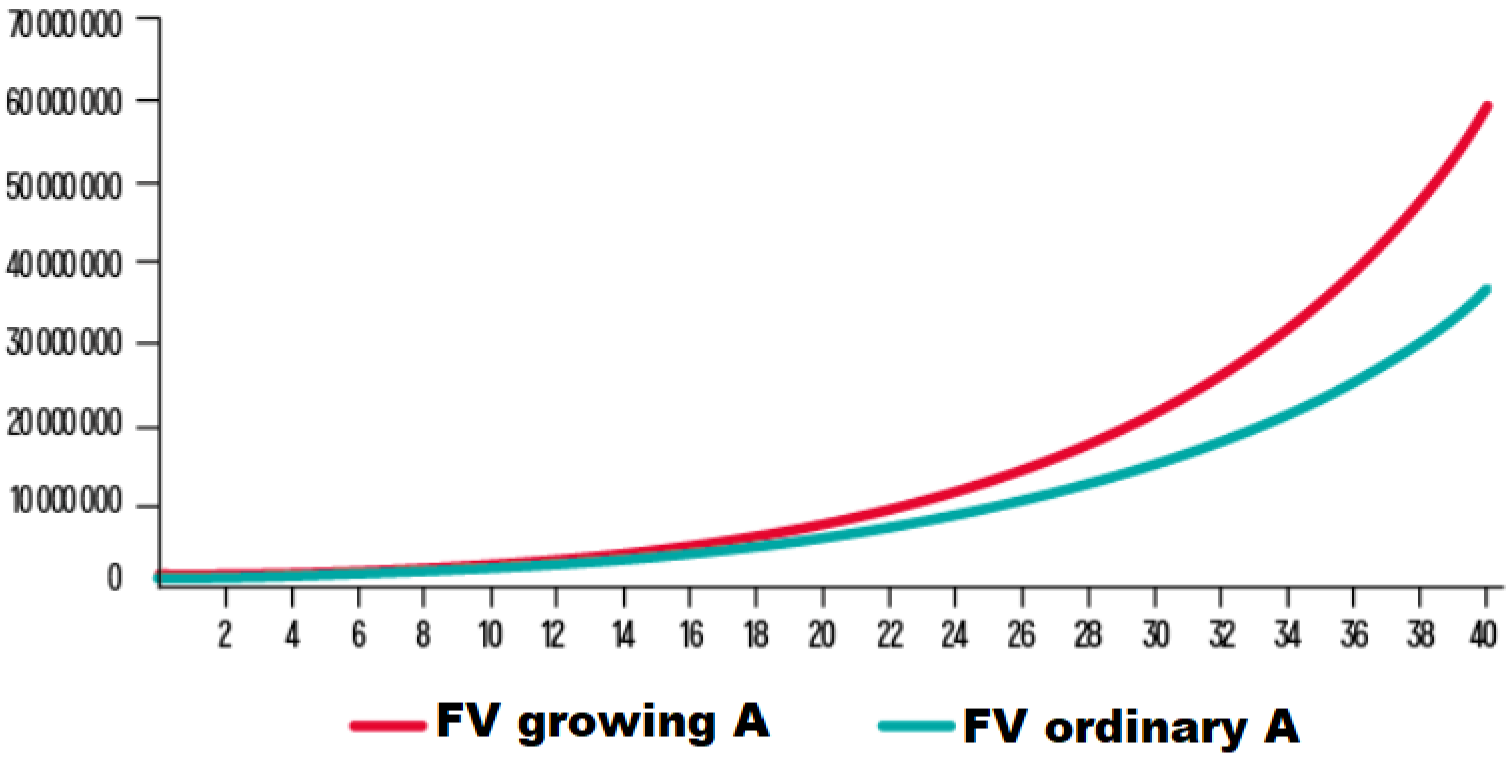


Growing vs. Ordinary Annuity

In the picture we see that we can really save more money using the growing annuity than just ordinary annuity.

When the growth will be approx. as the inflation, then we are able to save the purchasing power of our money.

Fight against inflation

1. You are going to start saving for your children's future studies. This year you will be able to save 5 thousand CZK. This amount will increase by 4% each year according to your calculations. The alternative cost is 8%. How much will you save in 15 years?

FV of a growing annuity

$$\boxed{FV = A\frac{(1+i)^n - (1+g)^n}{i-g}}$$

$$FV_{GA} = 5{,}000 * \frac{(1.08)^{15} - (1.04)^{15}}{0.08 - 0.04}$$

- Each other year You will save 4% more than the previous year.

Present value of growing annuity

2. What is the present value of your inheritance if your trustee is obliged to pay you a yearly starting amount of 10% annually starting this year for 10 years, while this year you will be paid 20 thousand USD (right now). We assume alternative costs of 11%.

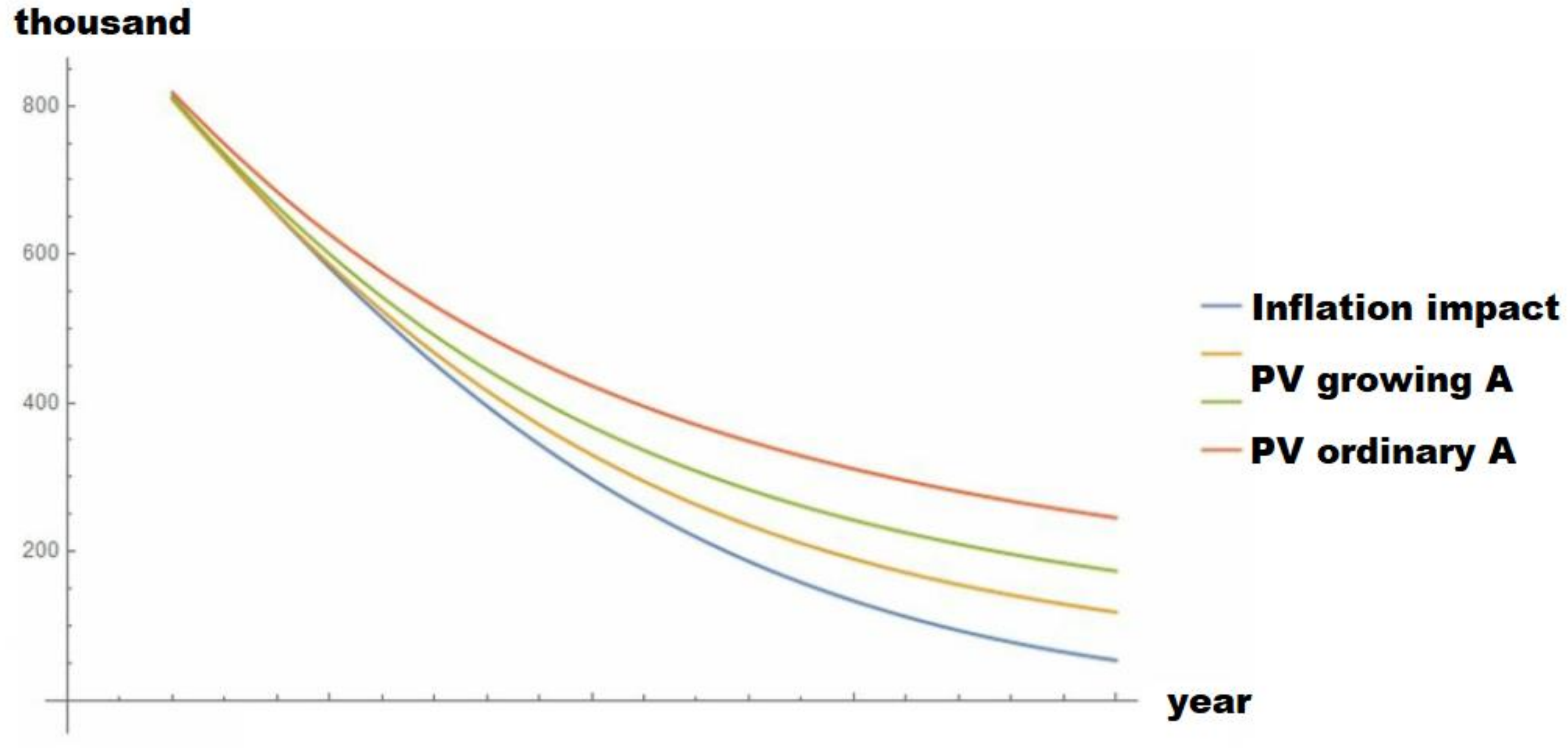
PV of a growing annuity

$$\boxed{PV = A\frac{1}{i-g}\left[1-\frac{(1+g)^n}{(1+i)^n}\right]}$$

$$PV_{GA} = \mathbf{20,000} + 22{,}000 * \frac{1}{0.11-0.10} * \left(1-\frac{1.10^{\mathbf{9}}}{1.11^{\mathbf{9}}}\right)$$

- It is still important remember what You have already learned! ☺

PV of growing annuity



PV of growing annuity

PV of growing annuity

The inflation can have really destructive impact on the purchasing power of our money. However, on the other hand, if we will withdraw more money each other year (growing A), we will have smaller amount than using the ordinary annuity from the point of view of present value.

Future value of growing annuity

3. You started saving for retirement this year. You are going to deposit 2% of your annual income every year, which is 150 thousand. CZK and will grow by 4% each year for the duration of your employment. You anticipate 8% savings per year. What amount will you have if you retire in 40 years?

FV of a growing annuity

$$\boxed{FV = A\frac{(1+i)^n - (1+g)^n}{i-g}}$$

$$FV_{GA} = 3{,}000 * \frac{(1.08)^{40} - (1.04)^{40}}{0.08 - 0.04}$$

- Please, be careful of the amount of annuity payment.

Two periods

4. What amount of funds will you dispose of in 2028, if you save 40 thousand CZK and next year by 4% more? While you will stop saving in 2026 (included), the money will be left in your account. Your account bears an interest rate of 6% p.a.

FV of a growing annuity

$$\boxed{FV = A\frac{(1+i)^n - (1+g)^n}{i-g}}$$

$$a)\ FV_{GA} = 40{,}000 * \frac{(1.06)^8 - (1.04)^8}{0.06 - 0.04}$$

$$b)\ FV = \boldsymbol{FV_{GA}} * (1.06)^2$$

- Combining of two periods is common ☺ CONTROLLING is necessary.

Be careful of a solution

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5. The couple saved up 1.5 million CZK. They want to use this money annually during their retirement over 20 years. The interest rate of such savings is 10% p.a. How much can they spend **next year** (first time) if they want to increase the amount by 5% p.a.?

PV of a growing annuity

$$\boxed{PV = A\frac{1}{i-g}\left[1-\frac{(1+g)^n}{(1+i)^n}\right]}$$

$$1{,}500{,}000 = A_{PV} * \frac{1}{0.10-0.05} * \left(1-\frac{1.05^{20}}{1.10^{20}}\right)$$

$$\boldsymbol{A_{PV}} = \frac{\boldsymbol{1,5mil * (0.10 - 0.05)}}{\left(\boldsymbol{1} - \frac{1.05^{20}}{1.10^{20}}\right)}$$

- A little more complicated one? ☺

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6. What is the present value of your heritage? Since this year, you will obtain a certain amount each year for 10 years. **This year** it is 25,000, - CZK and every next year by 10% more. The alternative cost is 12%.

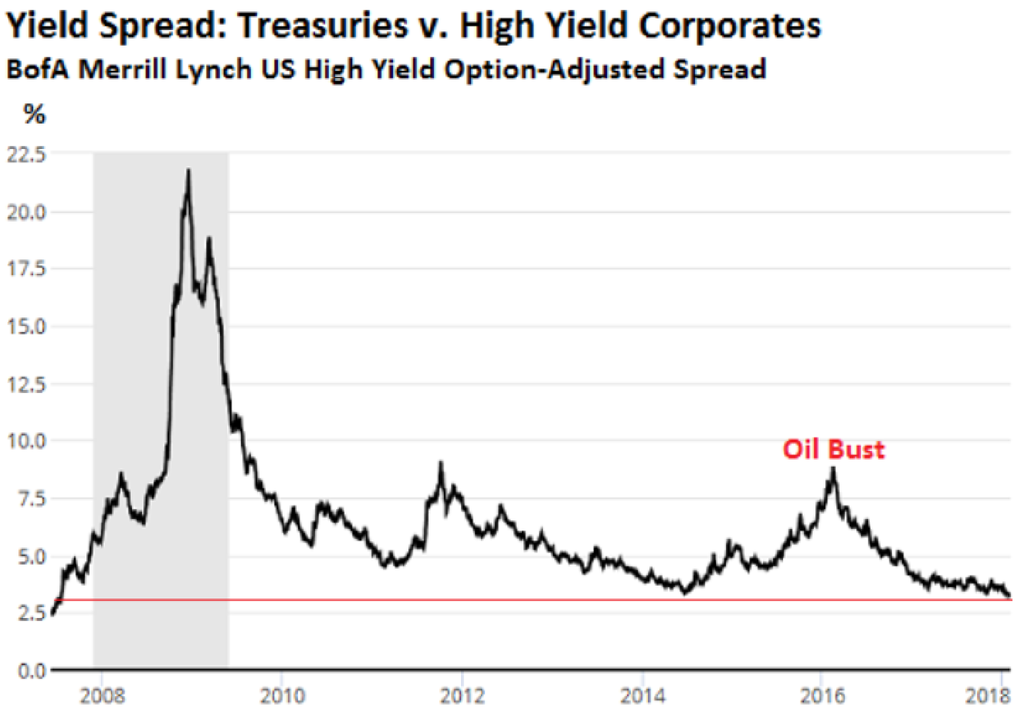
PV of a growing annuity

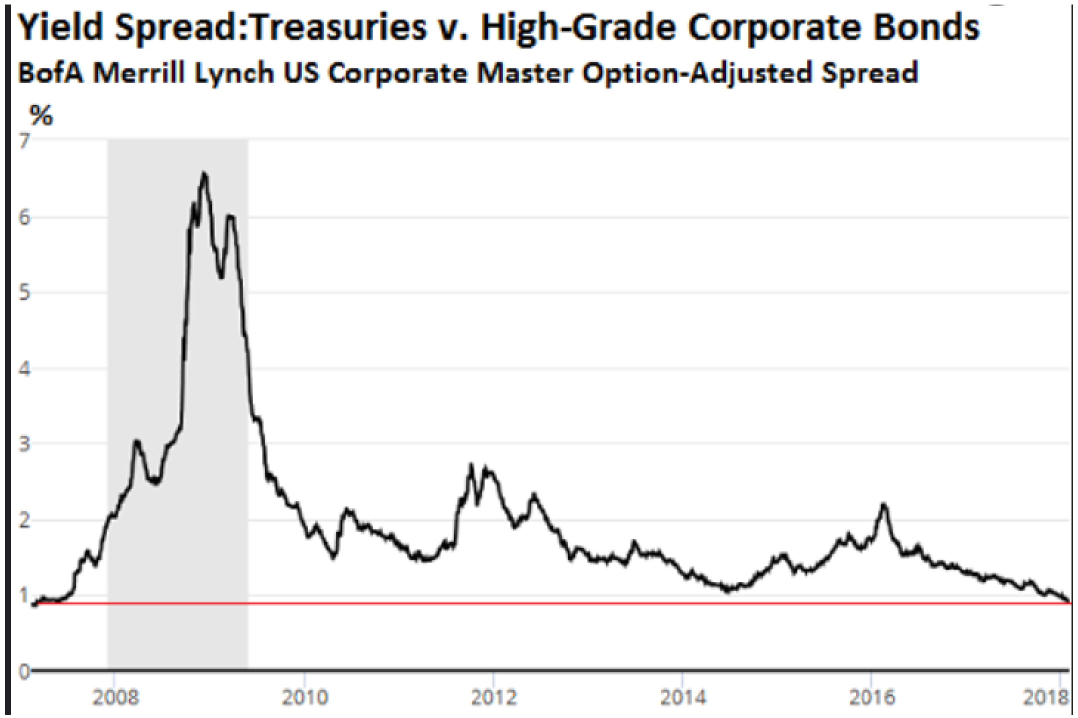
$$\boxed{PV = A\frac{1}{i-g}\left[1-\frac{(1+g)^n}{(1+i)^n}\right]}$$

$$PV_{GA} = \mathbf{25,000} + 27{,}500 * \frac{1}{0.12-0.10} * \left(1-\frac{1.10^{\mathbf{9}}}{1.12^{\mathbf{9}}}\right)$$

- However, this is a usual example ☺

Bond yields (JUNK vs. INVEST. GRADE)





Comparison of bond yields

Comparison of bond yields

Junk bonds are high-paying bonds with a lower credit rating than investment-grade corporate bonds, Treasury bonds, and municipal bonds. Junk bonds are typically rated 'BB' or lower by Standard & Poor's and 'Ba' or lower by Moody's. Despite their name, junk bonds can be valuable investments for informed investors, but their potential high returns come with the potential for high risk.

PV of perpetuity

A perpetuity is a security that pays for an infinite amount of time. In finance, perpetuity is a constant stream of identical cash flows with no end.

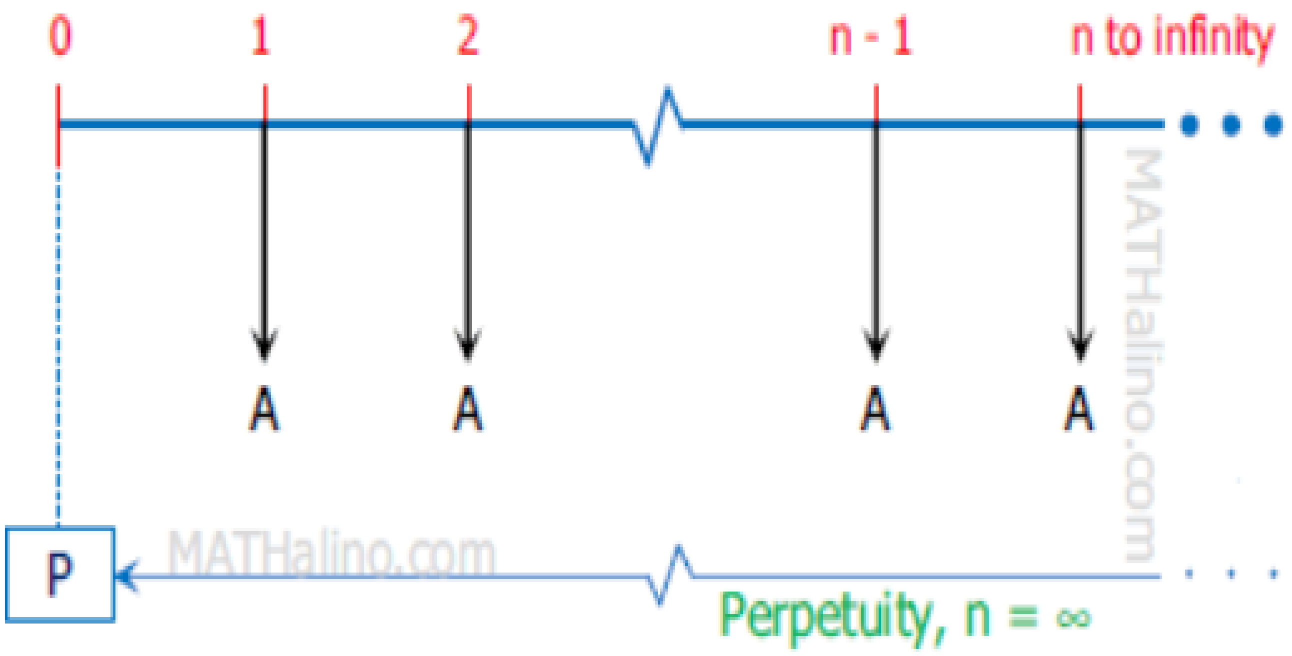
Specifically, the perpetuity formula determines the amount of cash flows in the terminal year of operation. In valuation, a company is said to be a going concern, meaning that it goes on forever.

An annuity is a stream of cash flows. A perpetuity is a type of annuity that lasts forever, into perpetuity. The stream of cash flows continues for an infinite amount of time.

In finance, a person uses the perpetuity calculation in valuation methodologies to find the present value of a company's cash flows when discounted back at a certain rate.

An example of a financial instrument with perpetual cash flows is the British-issued bonds known as consols. By purchasing a consol from the British government, the bondholder is entitled to receive annual interest payments forever.

Perpetuity means infinity



Cash flow diagram of perpetuity

Perpetuity means infinity

Because of the never ending period, FV of perpetuity does not exist! Therefore, we should calculate only PV.

Examples PERPETUITY

7. Calculate the market price of the share if the alternative cost is 5% and this year's dividend is100 CZK per share. The company's dividend policy is **stable**.

PV of an ordinary perpetuity

$$\boxed{PV = \frac{C}{i}}$$

$$PV_p = 100 + \frac{100}{0{,}05}$$

- The question is just whether the dividend was not already paid.

Examples of shares 1/4

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8. What is the market price of a share if DPS **a)** will be 100 CZK; **b)** was 100 CZK this year. We do expect DPS to increase by 10% each year and alternative costs of 15%?

Present value of a growing perpetuity

$$\boxed{PV = \frac{C}{i - g}}$$

$$a)\ PV_p = 100 + \frac{110}{0{,}15 - 0{,}10}$$

$$b)\ PV_p = \cancel{100 +} \frac{110}{0{,}15 - 0{,}10}$$

- If the dividend was already paid out we should count with the increased one.

Examples of shares **2/4**

9. What is the market price of a share if the dividend per share (DPS) **was** CZK 100 per share **last year**? We do expect DPS to increase by 10% each year and alternative costs of 15% this year and beyond?

Present value of a growing perpetuity

$$\boxed{PV = \frac{C}{i - g}}$$

$$PV_p = 110 + \frac{121}{0{,}15 - 0{,}10}$$

- You should calculate present value, so for the current year.

Examples of shares **3/4**

10. The company decided to pay a dividend of CZK 400 per share **next year**, with the dividend increasing by 4% per year. What is the current value of dividend income if the interest rate is 14%.

Present value of a growing perpetuity

$$\boxed{PV = \frac{C}{i - g}}$$

$$PV_p = \frac{400}{0{,}14 - 0{,}04}$$

- This kind of decision is made during board meeting when dividends have been already paid.

Examples of shares 4/4

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11. What should be the market value of one share of the company, if the **expected** profit this year is 5 million CZK, the company has 10 thousand of outstanding shares and shareholders will pay 50% of earnings in the form of dividends? The expected annual increase in profit is 5%. The alternative cost is 12%.

Present value of a growing perpetuity

$$\boxed{PV = \frac{C}{i - g}}$$

$$PV_P = 250 + \frac{262.5}{0.12 - 0.05}$$

- As the first You should calculate DPS 250,- CZK ☺

Perpetuity means forever

12. You own the land you rent for an indefinite period (forever). The land yield will increase annually by 5%. If the cash flow in the first year is CZK 10,000 and the interest rate is 10%, what is the value of the land?

Present value of a growing perpetuity

$$\boxed{PV = \frac{C}{i - g}}$$

$$PV_P = \mathbf{10,000} + \frac{10,500}{0.10 - 0.05}$$

- You should receive the rent right now and calculate with future cash flows.

Harder combinations 1/3

13. SINCE next year, you intend to deposit a certain amount each year up to 2024 (including next year as well as 2024). Then you want to withdraw CZK 12,000 each year from your account during the following years (2025 - 2041). If your account bears 2% p.a., how much do you have to deposit?

PV of an ordinary annuity

$$\boxed{PV = A\frac{(1+i)^n - 1}{i(1+i)^n}}$$

Annuity from FV

$$\boxed{A = FV\frac{i}{(1+i)^n - 1}}$$

$$PV_A = 12{,}000 * \frac{1.02^{17} - 1}{0.02 * 1.02^{17}}$$

$$A_{FV} = PV_A * \frac{0.02}{1{,}02^5 - 1}$$

- You have to have the money saved to withdraw 12,000 (**PV**). However, as the first think You have to save this money as FV from this certain amount (**A_{FV}**)

Harder combinations 2/3

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14. SINCE next year, you intend to deposit a certain amount each year up to 2024 (including next year as well as 2024). This year it will be 10,000, - CZK and every next year by 2% more. During the following years (2025 - 2030), how much will you be able to withdraw from your account each year? The account bears 3% p.a.

FV of a growing annuity

$$\boxed{FV = A\frac{(1+i)^n - (1+g)^n}{i-g}}$$

$\longrightarrow$

Annuity from PV

$$\boxed{A = PV\frac{(1+i)^n i}{(1+i)^n - 1}}$$

$$\boldsymbol{FV_A} = 10{,}000 * \frac{1.03^5 - 1{,}02^5}{0.03 - 0.02}$$

$$A_{PV} = \boldsymbol{FV_A}\frac{0.03 * 1.03^6}{1.03^6 - 1}$$

- As the first, You will save this 10,000 into the future (**FV**). Then You will withdraw certain amount from the money You have already saved (**A**PV)

Harder combinations 3/3

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15. You have won CZK 1 million in the lottery and you will save your winnings to the bank today. You will add CZK 100,000 annually to the same account for 10 years between 2021 and 2030. How much will you have in your account **at the beginning** of 2050? The account bears 3% p.a. at a **monthly basis**.

Effective annual interest rate

$$\boxed{EAIR = \left[1 + \frac{i}{m}\right]^m - 1}$$

FV of an ordinary annuity

$$\boxed{FV = A\frac{(1+i)^n - 1}{i}}$$

$$a)\ EAIR = (1 + \frac{0.03}{12})^{12} - 1$$

$$b)\ FV_A = 100{,}000 * \frac{(1 + EAIR)^{10} - 1}{EAIR}$$

$$FV = FV_A * (1 + EAIR)^{19}$$

$$c)\ FV = 1mil * (1 + EAIR)^{31}$$

- Three steps should be done ☺ Finally, b) PLUS c) is the solution.

Stock prices' development

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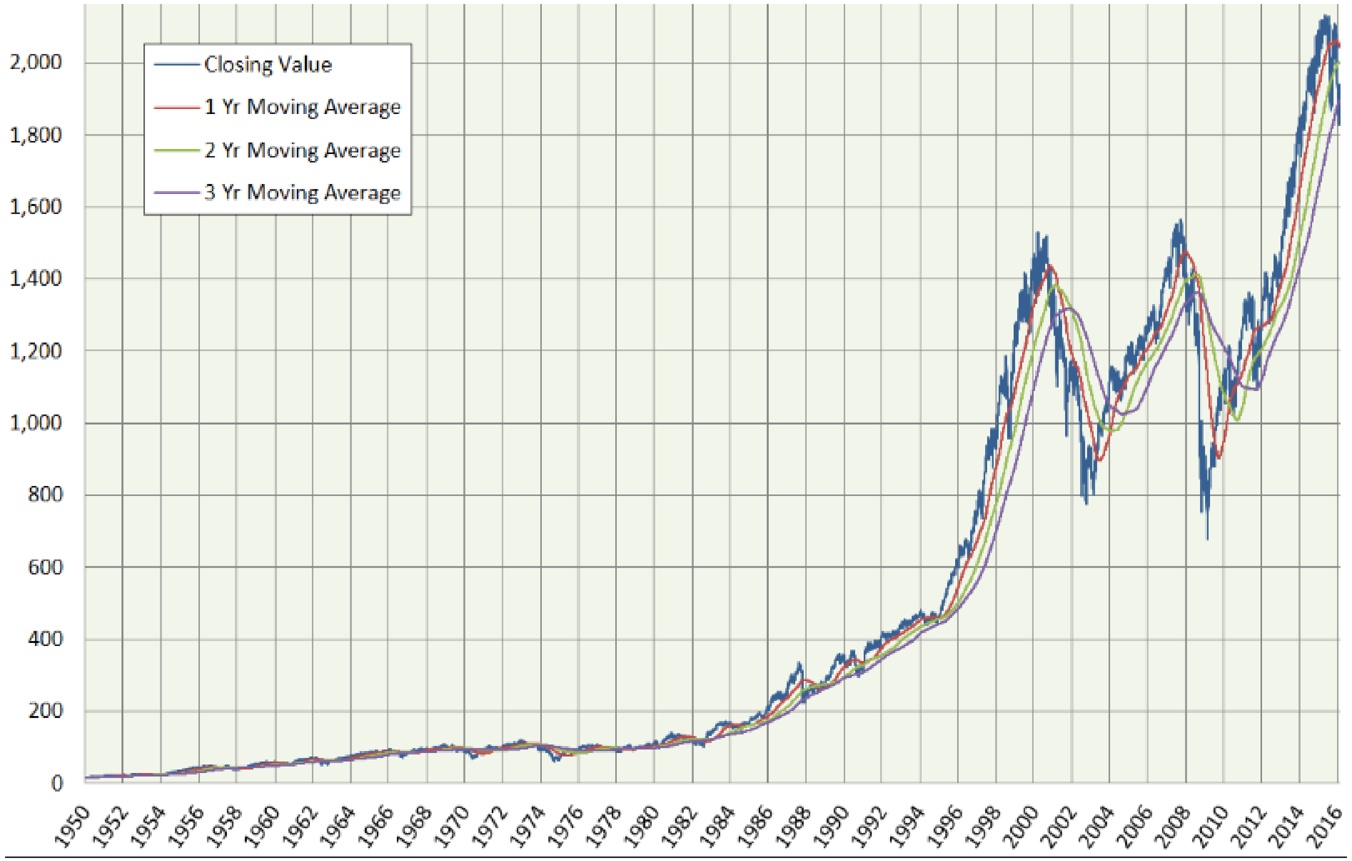

S&P 500

S&P 500

The S&P 500 index consists of most but not all of the largest companies in the United States. The S&P market cap is 70 to 80% of the total US stock market capitalization. It is a commonly used benchmark for stock portfolio performance in America and abroad.

Beating the performance of the S&P with less risk is the goal of nearly every portfolio manager, hedge fund and private investor.

Literature:

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3) BERK, J. & P. DeMARZO, 2017. The Time Value of Money. In: Corporate Finance, Chap. 4, pp. 130-174. ISBN 978-1-292-16016-0.

Thank you for
your attention!

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