Time value of money

Presentation for the Corporate Finance

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Content of this lecture

- Time value of the money
- Basics of the interest rates
- Basics related to bonds
- Few combinations

Present vs. Future value

The time value of money is a key concept in financial management, respectively corporate finance.

It is important for company's investments, financing and dividend decisions, which result in significant cash flows over a variety of time periods.

Imagine that somebody offers you 50 € now or 50 € in one year. Would you prefer money now or next year? You would choose probably money now for three main reasons:

The first one is time. You **prefer money now**, because you can spend it. You can buy whatever you want. You can invest it, if you do not want to spend it now and profit from it. In every case you take it now.

The second reason is influence of **inflation**. If you take money now, you can buy some certain amount of products, which you could not buy in one year, because they will be more expensive. Inflation causes increase of their price.

And the third reason is **risk**. If you take money now, nobody can take it back. It is yours instead of the situation that somebody promised you 50 € in one year and something would happen and you would not get them or at least not all of it.

Example FV

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1. What amount will you be able to withdraw from your bank account after 9 years if you deposit CZK 50,000 today and the deposit bears 2.5% p.a.?

Future value

$$FV = C_0(1+i)^n$$

$$FV = 50{,}000 * (1 + 0.025)^9$$

- It is much better to use directly 1.025 ☺
- However, never forget the ZERO with % in its absolute value!

Example PV

2. What is the present value of the investment, which after 15 years will yield a return of CZK 1 million? Alternative costs can be 8% p.a.

Present value

$$PV = \frac{C_n}{(1+i)^n}$$

- Again, 1.08 instead (1+0.08)! ☺

$$PV = \frac{1mil}{(1.08)^{15}}$$

Interest rate / average costs / yield ??

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3. The entrepreneur expects this year's profit of 500 thousand CZK. For the year would like to invest in new production technology 580 thousand CZK. At what yield can he realize his plan if the profit invested?

$$FV = PV * (1+i)^n \qquad PV = \frac{FV}{(1+i)^n}$$

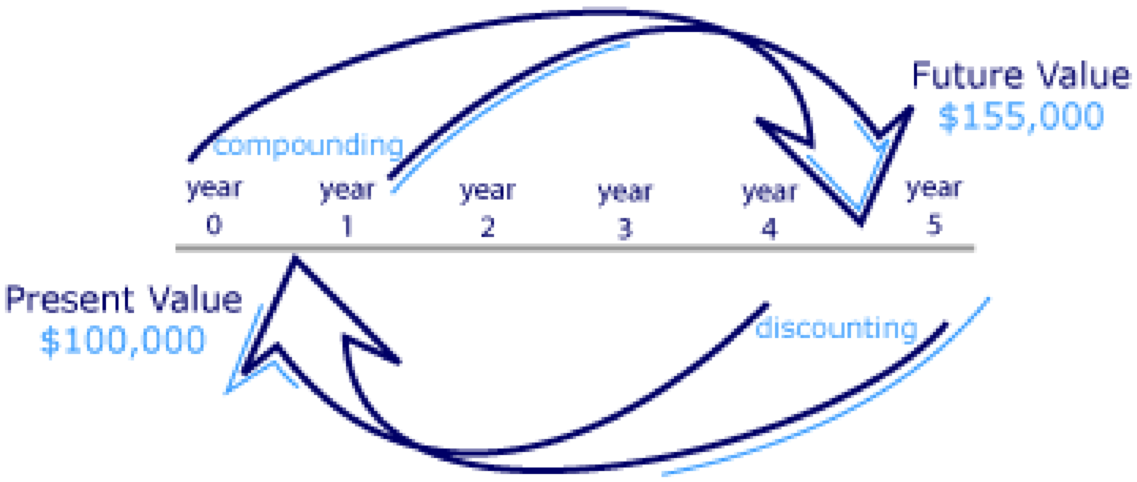
$$580{,}000 = 500{,}000 * (1+i)^1$$

- It is up to You which formulae You want to use to find out (i) ☺
- Please, be aware of a mistake with MINUS one!

$$(1+i) = \frac{580}{500}$$

$$i = \frac{580}{500} - 1$$

Time value of the money



Time value

Time value

The most important question is:

What are we going to calculate ??

The future or the present value?

The answer is always clear when we understand what we do know within an exercise.

The solution is always unknown then!

Example:

If we know future value we **cannot** calculate its future value again!

Example Bonds

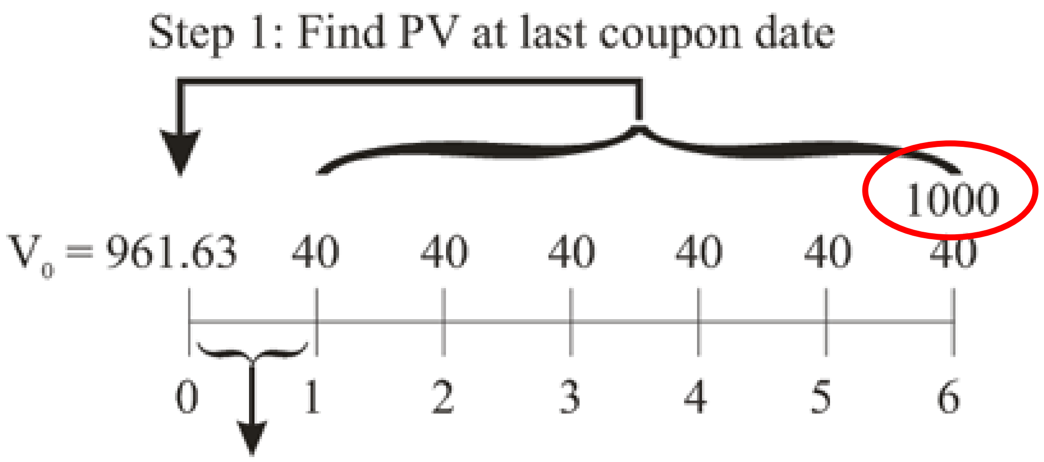
4. What is the present value of a bond with a nominal value of CZK 3,000, if you know that the yield to maturity is 6.7% p.a.? The annual coupon payment is 5.5% and the bond is payable in 5 years.

$$PV = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \cdots \frac{CF_n}{(1+i)^n} + \frac{P_0}{(1+i)^n}$$

$$PV = \frac{165}{(1.067)^1} + \frac{165}{(1.067)^2} + \cdots + \frac{165 + 3000}{(1.067)^5}$$

- Never forget on the final step – to discount a nominal value ☺

Explanation of the bond valuation



Time value: BONDS means PRESENT value!

BOND = Present value

In the case of the BOND, no matter what, its market value, price to buy/sell, it is always the PRESENT value!

All the time we would like to know it right now!

Furthermore, **we do know its future value** because of we are going to pay the debt back for its nominal value when its maturity expires.

Future or the Present? That's the question

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5. Compare the two following revenue if the alternative cost is 11% p.a .:

a) after 3 years you **will** receive CZK 10 million;

b) after 5 years you **will** receive CZK 20 million

$$PV = \frac{10mil}{(1.11)^3} \qquad PV = \frac{20mil}{(1.11)^5}$$

- Of course, we are taking into account just the revenue of the investment, not its liquidity nor its risk (**Magic triangle**).

Multi-period compounding

6. You want to deposit 50 thousand CZK to the bank for one year. One bank offers you a **Quarterly** interest rate of 2.8% p.a. and the second bank a **Continuous** compounding interest rate 2.7% p.a.

a) What is more advantageous for you?

b) What will be the situation in both banks after 2 and 3 years?

FV with multiple compounding

$$FV = C_0 (1 + \frac{i}{m})^{nm}$$

$$a)\ FV = 50{,}000 * \left(1 + \frac{0.028}{4}\right)^{1*4}$$

FV, continuous compounding

$$FV = C_0 (e^{in})$$

$$b)\ FV = 50{,}000 * 2.718^{0.027*1}$$

Interest vs. Discount rates

The preceding analysis is typical of **decision making in corporations** today, though real-world examples are, of course, much more complex.

Unfortunately, any example with risk poses a problem not faced in a riskless example. Conceptually, the **correct discount rate** for an expected cash flow is the expected return available in the market on other investments of the same risk.

This is the appropriate discount rate to apply because it represents an economic **opportunity cost to investors**. It is the expected return they will require before committing funding to a project.

Because the choice of a discount rate is so difficult, we merely wanted to broach the subject here.

We must wait until the specific material on risk and return is covered in later lectures before a risk-adjusted analysis can be presented.

Multi-period compounding with the rates

7. The Bank offers an interest rate on deposits of 3% p.a. Calculate the effective average interest rate if the interest is on:

a) annually

b) semi-annually

c) quarterly

d) monthly

e) daily

Effective annual interest rate

$$EAIR = \left[1 + \frac{i}{m}\right]^{m} - 1$$

$$a)\ EAIR_2 = \left(1 + \frac{0.03}{1}\right)^{1} - 1$$

$$b)\ EAIR_2 = \left(1 + \frac{0.03}{2}\right)^{2} - 1$$

$$c)\ EAIR_4 = \left(1 + \frac{0.03}{4}\right)^{4} - 1$$

$$d)\ EAIR_{12} = \left(1 + \frac{0.03}{12}\right)^{12} - 1$$

$$e)\ EAIR_{360} = \left(1 + \frac{0.03}{360}\right)^{360} - 1$$

Multiple-period compounding FV

8. What will be the value of 12 thousand CZK after 5 years for all those interest rates from the previous example?

FV with multiple compounding

$$FV = C_0 (1 + \frac{i}{m})^{nm}$$

$$a)\ FV_2 = 12{,}000 * \left(1 + \frac{0.03}{1}\right)^{5*1}$$

$$b)\ FV_2 = 12{,}000 * \left(1 + \frac{0.03}{2}\right)^{5*2}$$

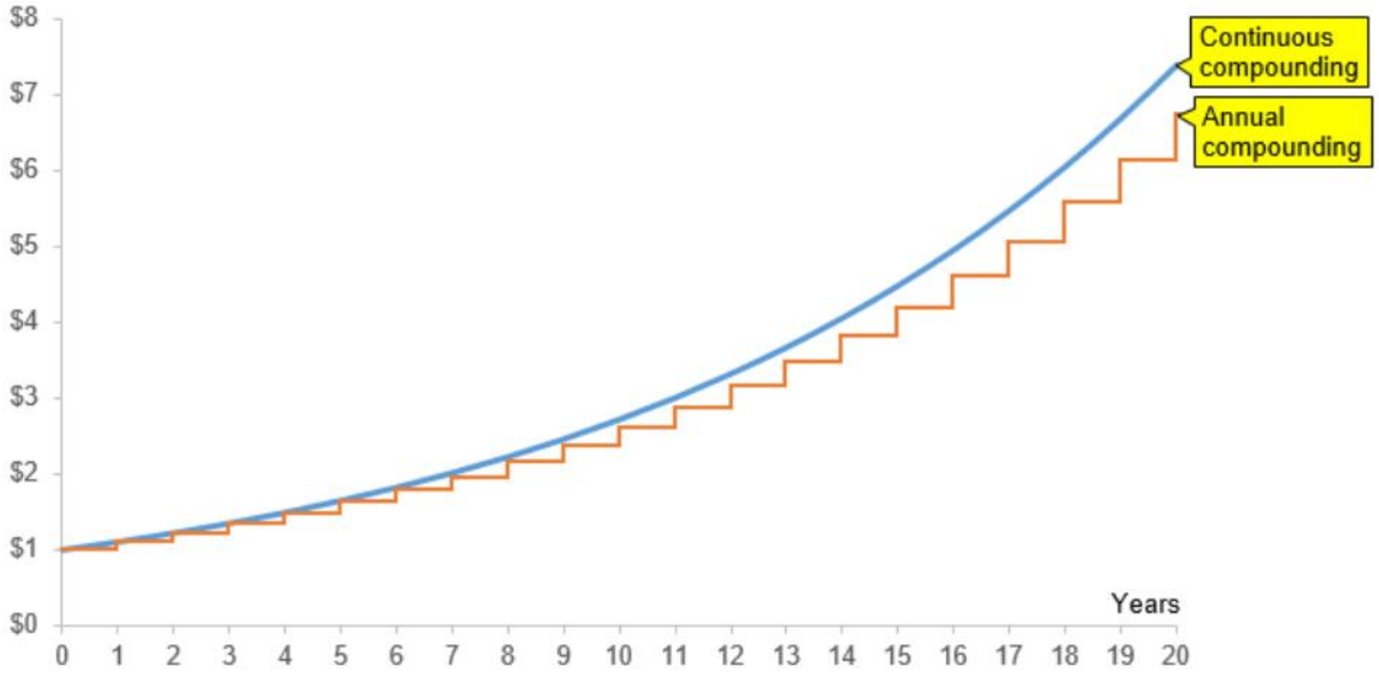
$$c)\ FV_4 = 12{,}000 * \left(1 + \frac{0.03}{4}\right)^{5*4}$$

$$d)\ FV_{12} = 12{,}000 * \left(1 + \frac{0.03}{12}\right)^{5*12}$$

$$e)\ FV_{360} = 12{,}000 * \left(1 + \frac{0.03}{360}\right)^{5*360}$$

- Surely, we can compound each interest rate and use that within the basic formulae. But this is a much more precise way to count this example ☺

Compounding



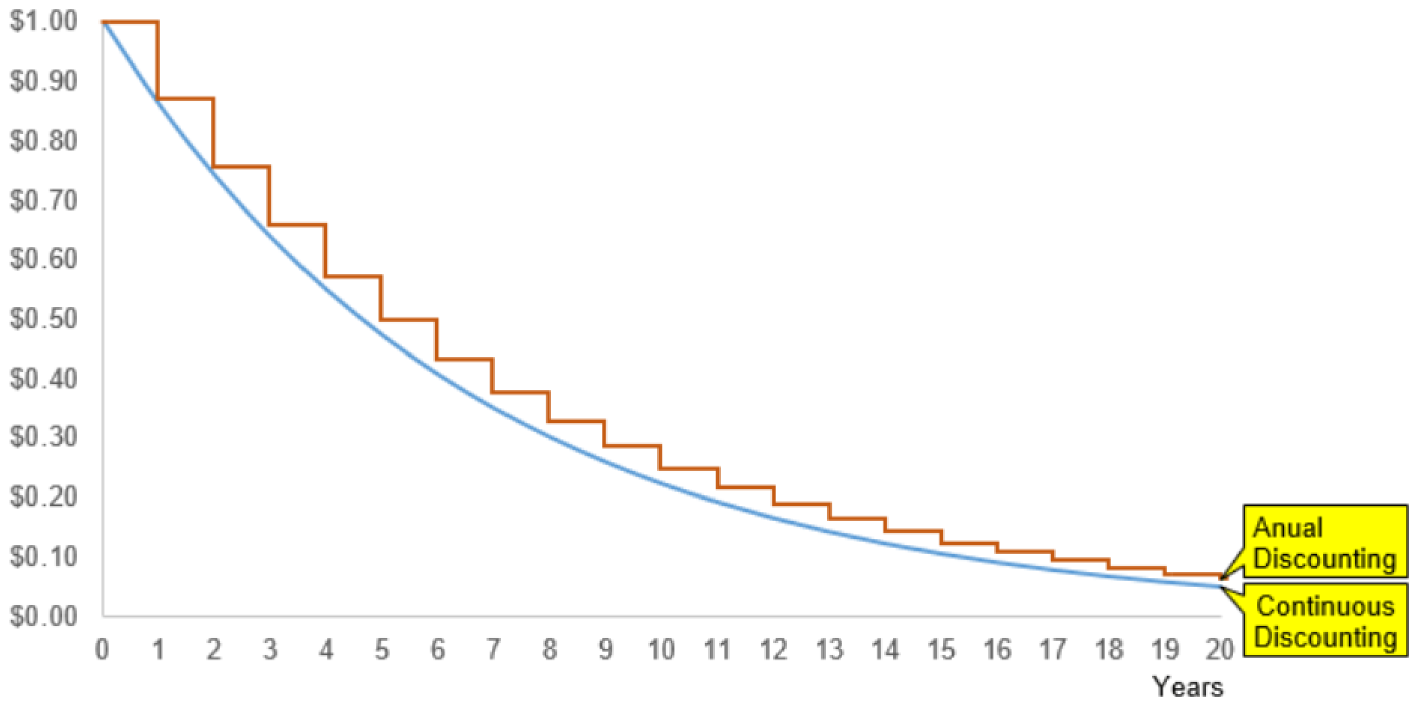
Time value: Compounding

Time value: Compounding

As we see in the picture if we compound more often (i.e. semi-annual, quarterly, monthly, daily, or even continuously), it means we are going to earn more money because of the interests earned.

GOOD FOR US! ☺

Discounting



Time value: Discounting

Time value: Discounting

However, on the other hand, when we are going to discount the money, or pay even the interests in the case of PV within a loan, more often discounting (i.e. continuous), it means higher costs in form of interests paid.

DEFINITELY NOT GOOD whether we are not a bank.

EAIR or p.a. compounding?

The distinction between the annual percentage rate, APR or p.a., and the effective annual interest rate (EAIR) is frequently troubling to students. We can reduce the confusion by noting that the APR becomes meaningful only if the compounding **interval is given**. (*e.g. 10Y*)

In other words, we do not know whether to compound semiannually, quarterly, or over some other interval. By contrast, the EAIR is meaningful **without** a compounding interval. (*e.g. 10Y again*)

There can be a big difference between an APR and an EAIR when interest rates are large. For example, consider "**payday loans**".

Payday loans are **short-term loans** made to consumers, often for less than two weeks. They are offered by companies such as *Check Into Cash* and *AmeriCash Advance*. The loans work like this: You write a check today that is postdated.

When the check date arrives, you go to the store and pay the cash for the check, or the company cashes the check. For example, in one particular state, *Check Into Cash* allows you to write a check **for $115 dated 14 days** in the future, for which **they give you $100 today**.

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Combining current knowledges 1/5

9. What will be the value of your deposit at the bank if this deposit bears 1.7% p.a. **two years**, and then another **four years quarterly**? The amount of the deposit is 90 thousand CZK.

$$FV_2 = 90{,}000 * (1.017)^{2} \quad \longrightarrow \quad FV_4 = FV_2\left(1 + \frac{0.017}{4}\right)^{4*4}$$

- Always think carefully before You start compounding! ☺

Combining current knowledges 2/5

11. The CNB treasury bill of nominal value 100 thousand CZK with a maturity of one year, the market price is CZK 89,000. Calculate the present value Factor and the corresponding alternative cost.

$$100{,}000 = 89{,}000 * (1+i)^1$$

$$(1+i)^1 = \frac{100}{89}$$

$$i = \frac{100}{89} - 1$$

- PV factor is just:

$$PV_f = \frac{1}{(1+i)}$$

Combining current knowledges 3/5

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12. Imagine you won 100,000 CZK today in a lottery. You are going to save this money on your bank account and buy a used car in 5 years. You assume that you need CZK 161,050 at that time. What is the interest rate for your account? Is this rate a real one nowadays?

$$161{,}000 = 100{,}000 * (1+i)^5$$

$$(\mathbf{1}+\boldsymbol{i})^{\mathbf{5}} = \frac{161}{100}$$

$$\boldsymbol{i} = \sqrt[\mathbf{5}]{\frac{161}{100}}\,(-\mathbf{1})$$

- **REAL** ???

Nominal vs. Real GDP ???

Combining current knowledges 4/5

13. Mrs. ZHANG plans to buy lucrative land around the future highway. The land is now for sale for CZK 8.5 million and experts expect the price to rise to around CZK 9.1 million **in one year**. This would amount to a profit of CZK 600,000.

a) Is such an investment advantageous if she can invest money with a yield of 10% p.a.

b) What would the interest rate have to be in order for her to make a profit of 600 thousand CZK?

$$a)\ FV_{investment} = 8.5mil * (1.1)^{1}$$

$$b)\ 9{,}5mil = 8{,}5mil * (1+i)^{1}$$

$$(1+i)^{1} = \frac{9{,}5}{8{,}5}$$

$$i = \frac{0{,}6}{8{,}5}$$

$$i = \frac{9{,}5}{8{,}5} - 1$$

- **Only in few cases** it is up to You, whether You use the formulae or just the logic ☺

Combining current knowledges 5/5

10. What is the **effective average interest rate** if the account is **continuously compounded** at 2.7%?

FV with multiple compounding

$$FV = C_0 \left(1 + \frac{i}{m}\right)^{nm}$$

→

FV, continuous compounding

$$FV = C_0 \left(e^{in}\right)$$

→

Effective annual interest rate

$$EAIR = \left[1 + \frac{i}{m}\right]^{m} - 1$$

$$EAIR_e = 2{,}718^{0{,}027*1} - 1$$

- It seems to be a simple one, doesn't it? ☺

Summarizing

The first lecture of this course has examined the concepts of future value and present value.

Although these concepts allow us to answer many problems concerning the time value of money, the effort involved can be excessive.

Consider a bank calculating the present value of a **20-year monthly mortgage**—or **annuity payments** of mortgage loans, **stocks**, etc.

Because many basic finance problems are potentially time-consuming, we will search for simplifications in a few future lectures.

We will provide simplifying formulas and examples for four classes of cash flow streams:

- **Perpetuity**
- **Growing perpetuity**
- **Annuity**
- **Growing annuity**

Literature

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1) Seminar 01 Examples
2) ROSS, S. A., R. W. WESTERFIELD, J. JAFFE & B. D. JORDAN, 2019. Valuation and Capital Budgeting. In: *Corporate Finance*, PART II, pp. 85-298. ISBN 978-1-260-09187-8.
3) BERK, J. & P. DeMARZO, 2017. The Time Value of Money. In: Corporate Finance, Chap. 4, pp. 130-174. ISBN 978-1-292-16016-0.

- Please, use Your own calculator…not a smart phone or anything smart instead Your brain! ☺

Thank you for your attention!

☺