**Conditional Statement**

**Definition: A Conditional Statement is...**

symbolized by phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifq, it is an if-then statement in which p is a [hypothesis](javascript:popUpWindow('hypothesis')) and q is a [conclusion](javascript:popUpWindow('conclusion')). The logical connector in a conditional statement is denoted by the symbol https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gif. The conditional is defined to be true unless a true hypothesis leads to a false conclusion. A truth table for phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifq is shown below.

|  |  |  |
| --- | --- | --- |
| p | q | phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifq |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

In the truth table above, phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifq is only false when the hypothesis (p) is true and the conclusion (q) is false; otherwise it is true. Note that a conditional is a [compound statement](javascript:popUpWindow('compound_statement')). Now that we have defined a conditional, we can apply it to Example 1.

Example 1:

|  |  |
| --- | --- |
| Given: | p: I do my homework. |
| q: I get my allowance. |
| Problem: | What does phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifq represent? |

Solution: In Example 1, the sentence, "I do my homework" is the hypothesis and the sentence, "I get my allowance" is the conclusion. Thus, the conditional phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifq represents the hypothetical proposition, "If I do my homework, then I get an allowance." However, as you can see from the truth table above, doing your homework does not guarantee that you will get an allowance! In other words, there is not always a cause-and-effect relationship between the hypothesis and conclusion of a conditional statement.

Example 2:

|  |  |
| --- | --- |
| Given: | a: The sun is made of gas. |
| b: 3 is a prime number. |
| Problem: | Write ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifb as a sentence. Then construct a truth table for this conditional. |

Solution: The conditional ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifb represents "If the sun is made of gas, then 3 is a prime number."

|  |  |  |
| --- | --- | --- |
| a | b | ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifb |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

In Example 2, "The sun is made of gas" is the hypothesis and "3 is a prime number" is the conclusion. Note that the logical meaning of this conditional statement is not the same as its intuitive meaning. In logic, the conditional is defined to be true unless a true hypothesis leads to a false conclusion. The implication of ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifb is that: since the sun is made of gas, this makes 3 a prime number. However, intuitively, we know that this is false because the sun and the number three have nothing to do with one another! Therefore, the logical conditional allows implications to be true even when the hypothesis and the conclusion have no logical connection.

Example 3:

|  |  |
| --- | --- |
| Given: | x: Gisele has a math assignment. |
| y: David owns a car. |
| Problem: | Write xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gify as a sentence. |

Solution: The conditional xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gify represents, "If Gisele has a math assignment, then David owns a car.

**In the following examples, we are given the truth values of the hypothesis and the conclusion and asked to determine the truth value of the conditional.**

Example 4:

|  |  |  |
| --- | --- | --- |
| Given: | r: 8 is an odd number. | false |
| s: 9 is composite. | true |
| Problem: | What is the truth value of  rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifs? | |

Solution: Since hypothesis r is false and conclusion s is true, the conditional rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifs is true.

Example 5:

|  |  |  |
| --- | --- | --- |
| Given: | r: 8 is an odd number. | false |
| s: 9 is composite. | true |
| Problem: | What is the truth value of  shttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifr? | |

Solution: Since hypothesis s is true and conclusion r is false, the conditional shttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifr is false.

Example 6:

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| --- | --- | --- |
| Given: | p: 72 = 49. | true |
| q: A rectangle does not have 4 sides. | false |
| r: Harrison Ford is an American actor. | true |
|  | s: A square is not a quadrilateral. | false |
| Problem: | Write each conditional below as a sentence. Then indicate its truth value. | |

|  |  |  |  |
| --- | --- | --- | --- |
| 1. | phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifq | If 72 is equal to 49, then a rectangle does not have 4 sides. | false |
| 2. | qhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifr | If a rectangle does not have 4 sides, then Harrison Ford is an American actor. | true |
| 3. | phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifr | If 72 is equal to 49, then Harrison Ford is an American actor. | true |
| 4. | qhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifs | If a rectangle does not have 4 sides, then a square is not a quadrilateral. | true |
| 5. | rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gif~p | If Harrison Ford is an American actor, then 72 is not equal to 49. | false |
| 6. | ~rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifp | If Harrison Ford is not an American actor, then 72 is equal to 49. | true |

Note that in item 5, the conclusion is the negation of p. Also, in item 6, the hypothesis is the negation of r.

**Summary:**A conditional statement, symbolized by phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional.gifq, is an if-then statement in which p is a hypothesis and q is a conclusion. The conditional is defined to be true unless a true hypothesis leads to a false conclusion.

**Exercises**

Directions: Read each question below. Select your answer by clicking on its button. Feedback to your answer is provided in the RESULTS BOX. If you make a mistake, choose a different button.

|  |  |
| --- | --- |
| **1.** | **Which of the following is a conditional statement?** |
|  | Začátek formuláře  Amy plays soccer or Bill plays hockey. Bill plays hockey when Amy plays soccer. If Amy plays soccer then Bill plays hockey. None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **2.** | |  |  | | --- | --- | | **Given:** | **r: You give me twenty dollars.** | | **s: I will be your best friend.** | | **Problem:** | **Which of the following statements represents, "If you give me twenty dollars, then I will be your best friend"?** | |
|  | Začátek formuláře  rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/and.gifs rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional_transp.gifs shttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional_transp.gifr None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |
| --- | --- |
| **3.** | **What is the truth value of r**https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional_transp.gif**s when the hypothesis is false and the conclusion is true in Example 2?** |
|  | Začátek formuláře  True False Not enough information was given. None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **4.** | |  |  | | --- | --- | | **Given:** | **a: x is prime.** | | **b: x is odd.** | | **Problem:** | **What is the truth value of a**https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional_transp.gif**b when x = 2?** | |
|  | Začátek formuláře  True False Not enough information was given. None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |
| --- | --- |
| **5.** | **What is the truth value of a**https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/conditional_transp.gif**b when x = 9 in Exercise 4?** |
|  | Začátek formuláře  True False Not enough information was given. None of the above.  RESULTS BOX:  Konec formuláře |