

Conditional Statement

Definition: A Conditional Statement is...

symbolized by $p \rightarrow q$, it is an **if-then statement** in which p is a [hypothesis](#) and q is a [conclusion](#). The logical connector in a conditional statement is denoted by the symbol \rightarrow . The conditional is defined to be true unless a true hypothesis leads to a false conclusion. A truth table for $p \rightarrow q$ is shown below.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

In the truth table above, $p \rightarrow q$ is only false when the hypothesis (p) is true and the conclusion (q) is false; otherwise it is true. Note that a conditional is a [compound statement](#). Now that we have defined a conditional, we can apply it to Example 1.

Example 1:

Given:	p: I do my homework.
	q: I get my allowance.
Problem:	What does $p \rightarrow q$ represent?

Solution: In Example 1, the sentence, "I do my homework" is the hypothesis and the sentence, "I get my allowance" is the conclusion. Thus, the conditional $p \rightarrow q$ represents the hypothetical proposition, "If I do my homework, then I get an allowance." However, as you can see from the truth table above, doing your homework does not guarantee that you will get an allowance! In other words, there is not always a cause-and-effect relationship between the hypothesis and conclusion of a conditional statement.

Example 2:

Given:	a: The sun is made of gas.
	b: 3 is a prime number.
Problem:	Write $a \rightarrow b$ as a sentence. Then construct a truth table for this conditional.

Solution: The conditional $a \rightarrow b$ represents "If the sun is made of gas, then 3 is a prime number."

a	b	$a \rightarrow b$
T	T	T

T	F	F
F	T	T
F	F	T

In Example 2, "The sun is made of gas" is the hypothesis and "3 is a prime number" is the conclusion. Note that the logical meaning of this conditional statement is not the same as its intuitive meaning. In logic, the conditional is defined to be true unless a true hypothesis leads to a false conclusion. The implication of $a \rightarrow b$ is that: since the sun is made of gas, this makes 3 a prime number. However, intuitively, we know that this is false because the sun and the number three have nothing to do with one another! Therefore, the logical conditional allows implications to be true even when the hypothesis and the conclusion have no logical connection.

Example 3:

Given:	x: Gisele has a math assignment.
	y: David owns a car.
Problem:	Write $x \rightarrow y$ as a sentence.

Solution: The conditional $x \rightarrow y$ represents, "If Gisele has a math assignment, then David owns a car."

In the following examples, we are given the truth values of the hypothesis and the conclusion and asked to determine the truth value of the conditional.

Example 4:

Given:	r: 8 is an odd number.	false
	s: 9 is composite.	true
Problem:	What is the truth value of $r \rightarrow s$?	

Solution: Since hypothesis r is false and conclusion s is true, the conditional $r \rightarrow s$ is true.

Example 5:

Given:	r: 8 is an odd number.	false
	s: 9 is composite.	true
Problem:	What is the truth value of $s \rightarrow r$?	

Solution: Since hypothesis s is true and conclusion r is false, the conditional $s \rightarrow r$ is false.

Example 6:

Given:	p: $7^2 = 49$.	true
	q: A rectangle does not have 4 sides.	false
	r: Harrison Ford is an American actor.	true

	s: A square is not a quadrilateral.	false
Problem:	Write each conditional below as a sentence. Then indicate its truth value.	
1.	$p \rightarrow q$ If 7^2 is equal to 49, then a rectangle does not have 4 sides.	false
2.	$q \rightarrow r$ If a rectangle does not have 4 sides, then Harrison Ford is an American actor.	true
3.	$p \rightarrow r$ If 7^2 is equal to 49, then Harrison Ford is an American actor.	true
4.	$q \rightarrow s$ If a rectangle does not have 4 sides, then a square is not a quadrilateral.	true
5.	$r \rightarrow \sim p$ If Harrison Ford is an American actor, then 7^2 is not equal to 49.	false
6.	$\sim r \rightarrow p$ If Harrison Ford is not an American actor, then 7^2 is equal to 49.	true

Note that in item 5, the conclusion is the negation of p. Also, in item 6, the hypothesis is the negation of r.

Summary: A conditional statement, symbolized by $p \rightarrow q$, is an if-then statement in which p is a hypothesis and q is a conclusion. The conditional is defined to be true unless a true hypothesis leads to a false conclusion.

Exercises

Directions: Read each question below. Select your answer by clicking on its button. Feedback to your answer is provided in the RESULTS BOX. If you make a mistake, choose a different button.

1. Which of the following is a conditional statement?

- Amy plays soccer or Bill plays hockey.
- Bill plays hockey when Amy plays soccer.
- If Amy plays soccer then Bill plays hockey.
- None of the above.

RESULTS BOX:

2. Given:	r: You give me twenty dollars.
	s: I will be your best friend.
Problem:	Which of the following statements represents, "If you give me twenty dollars, then I will be your best friend"?

- $r \wedge s$
- $r \rightarrow s$
- $s \rightarrow r$
- None of the above.

RESULTS BOX:

3. What is the truth value of $r \rightarrow s$ when the hypothesis is false and the conclusion is true in Example 2?

- True
- False
- Not enough information was given.
- None of the above.

RESULTS BOX:

4. Given:	a: x is prime.
	b: x is odd.
Problem:	What is the truth value of $a \rightarrow b$ when $x = 2$?

- True
- False
- Not enough information was given.
- None of the above.

RESULTS BOX:

5. What is the truth value of $a \rightarrow b$ when $x = 9$ in Exercise 4?

- True
- False
- Not enough information was given.
- None of the above.

RESULTS BOX: