**Disjunction**

Example 1:

|  |  |
| --- | --- |
| Given: | p: Ann is on the softball team. |
| q: Paul is on the football team. |
| Problem: | What does phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq represent? |

Solution: In Example 1, statement p represents, "Ann is on the softball team" and statement q represents, "Paul is on the football team." The symbol https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gif is a logical connector which means "or." Thus, the compound statement phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq represents the sentence, "Ann is on the softball team or Paul is on the football team." The statement phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq is a disjunction.

**Definition:**A **disjunction** is a compound statement formed by joining two statements with the connector OR. The disjunction "p or q" is symbolized by phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq. A disjunction is false if and only if both statements are false; otherwise it is true. The truth values of phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq are listed in the truth table below.

|  |  |  |
| --- | --- | --- |
| p | q | phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Example 2:

|  |  |
| --- | --- |
| Given: | a: A square is a quadrilateral. |
| b: Harrison Ford is an American actor. |
| Problem: | Construct a truth table for the disjunction "a or b." |

Solution:

|  |  |  |
| --- | --- | --- |
| a | b | ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifb |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Example 3:

|  |  |
| --- | --- |
| Given: | r: x is divisible by 2. |
| s: x is divisible by 3. |
| Problem: | What are the truth values of rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifs? |

Solution: Each statement given in this example represents an [open sentence](javascript:popUpWindow('open_sentence')), so the truth value of rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifs will depend on the replacement values of x as shown below.

If x = 6, then r is true, and s is true. The disjunction rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifs is true.

If x = 8, then r is true, and s is false. The disjunction rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifs is true.

If x = 15, then r is false, and s is true. The disjunction rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifs is true.

If x = 11, then r is false, and s is false. The disjunction rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifs is false.

Example 4:

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| --- | --- | --- |
| Given: | p: 12 is prime. | false |
| q: 17 is prime. | true |
| r: 19 is composite. | false |
| Problem: | Write a sentence for each disjunction below. Then indicate if it is true or false. | |

|  |  |  |  |
| --- | --- | --- | --- |
| 1. | phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq | 12 is prime or 17 is prime. | true |
| 2. | phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifr | 12 is prime or 19 is composite. | false |
| 3. | qhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifr | 17 is prime or 19 is composite. | true |

Example 5: Complete a truth table for each disjunction below.

1.  a or b

2.  a or not b

3.  not a or b

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  | | --- | --- | --- | | a | b | ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifb | | T | T | T | | T | F | T | | F | T | T | | F | F | F | | https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/tab.gif | |  |  |  |  | | --- | --- | --- | --- | | a | b | ~b | ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gif~b | | T | T | F | T | | T | F | T | T | | F | T | F | F | | F | F | T | T | | https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/tab.gif | |  |  |  |  | | --- | --- | --- | --- | | a | b | ~a | ~ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifb | | T | T | F | T | | T | F | F | F | | F | T | T | T | | F | F | T | T | |

Students sometimes confuse conjunction and disjunction. Let's look at an example in which we compare the truth values of both of these compound statements.

Example 6:

|  |  |
| --- | --- |
| Given: | x: Jayne played tennis. |
| y: Chris played softball. |
| Problem: | Construct a truth table for conjunction "x and y" and disjunction "x or y." |

Solution:

|  |  |  |  |
| --- | --- | --- | --- |
| x | y | xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/and.gify | xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gify |
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | F |

With a conjunction, **both statements must be true for the conjunction to be true**; but with a disjunction, **both statements must be false for the disjunction to be false**. One way to remember this is with the following mnemonic: 'And’ points up to the sand on top of the beach, while ‘or’ points down to the ore deep in the ground.

**Summary:** A **disjunction** is a compound statement formed by joining two statements with the connector OR. The disjunction "p or q" is symbolized by phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq. A disjunction is false if and only if both statements are false; otherwise it is true.

**Exercises**

Directions: Read each question below. Select your answer by clicking on its button. Feedback to your answer is provided in the RESULTS BOX. If you make a mistake, choose a different button.

|  |  |
| --- | --- |
| **1.** | **Which of the following sentences is a disjunction?** |
|  | Začátek formuláře  Amy played soccer or Bill played hockey. Amy played soccer and Bill played hockey. Amy did not play soccer and Bill played hockey. None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |
| --- | --- |
| **2.** | **Which of the following statements is a disjunction?** |
|  | Začátek formuláře  ~xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/and.gify xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/and.gify xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gify None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |
| --- | --- |
| **3.** | **A disjunction is used with which connector?** |
|  | Začátek formuláře  And Or Not None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |
| --- | --- |
| **4.** | **If a is false and b is true, what is the truth value of a**https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gif~**b?** |
|  | Začátek formuláře  True False Not enough information was given None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **5.** | |  |  | | --- | --- | | **Given:** | **r: y is prime.** | | **s: y is even.** | | **Problem:** | **Which of the following is a true statement when y is replaced by 3?** | |
|  | Začátek formuláře  rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gif~s rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/and.gif~s rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifs All of the above.  RESULTS BOX:  Konec formuláře |