**Disjunction**

Example 1:

|  |  |
| --- | --- |
| Given: | p: Ann is on the softball team. |
| q: Paul is on the football team. |
| Problem: | What does phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq represent? |

Solution: In Example 1, statement p represents, "Ann is on the softball team" and statement q represents, "Paul is on the football team." The symbol  is a logical connector which means "or." Thus, the compound statement pq represents the sentence, "Ann is on the softball team or Paul is on the football team." The statement pq is a disjunction.

**Definition:**A **disjunction** is a compound statement formed by joining two statements with the connector OR. The disjunction "p or q" is symbolized by pq. A disjunction is false if and only if both statements are false; otherwise it is true. The truth values of pq are listed in the truth table below.

|  |  |  |
| --- | --- | --- |
| p | q | phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Example 2:

|  |  |
| --- | --- |
| Given: | a: A square is a quadrilateral. |
| b: Harrison Ford is an American actor. |
| Problem:   | Construct a truth table for the disjunction "a or b." |

Solution:

|  |  |  |
| --- | --- | --- |
| a | b | ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifb |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Example 3:

|  |  |
| --- | --- |
| Given: | r: x is divisible by 2. |
| s: x is divisible by 3. |
| Problem: | What are the truth values of rhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifs? |

Solution: Each statement given in this example represents an open sentence, so the truth value of rs will depend on the replacement values of x as shown below.

If x = 6, then r is true, and s is true. The disjunction rs is true.

If x = 8, then r is true, and s is false. The disjunction rs is true.

If x = 15, then r is false, and s is true. The disjunction rs is true.

If x = 11, then r is false, and s is false. The disjunction rs is false.

Example 4:

|  |  |  |
| --- | --- | --- |
| Given: | p: 12 is prime. | false |
| q: 17 is prime. | true |
| r: 19 is composite. | false |
| Problem: | Write a sentence for each disjunction below. Then indicate if it is true or false. |

|  |  |  |  |
| --- | --- | --- | --- |
| 1. | phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifq | 12 is prime or 17 is prime. | true |
| 2. | phttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifr | 12 is prime or 19 is composite. | false |
| 3. | qhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifr | 17 is prime or 19 is composite. | true |

Example 5: Complete a truth table for each disjunction below.

1.  a or b

2.  a or not b

3.  not a or b

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| a | b | ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifb |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

 | https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/tab.gif |

|  |  |  |  |
| --- | --- | --- | --- |
| a | b | ~b | ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gif~b |
| T | T | F | T |
| T | F | T | T |
| F | T | F | F |
| F | F | T | T |

 | https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/tab.gif |

|  |  |  |  |
| --- | --- | --- | --- |
| a | b | ~a | ~ahttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifb |
| T | T | F | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

 |

Students sometimes confuse conjunction and disjunction. Let's look at an example in which we compare the truth values of both of these compound statements.

Example 6:

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| --- | --- |
| Given: | x: Jayne played tennis. |
| y: Chris played softball. |
| Problem: | Construct a truth table for conjunction "x and y" and disjunction "x or y." |

Solution:

|  |  |  |  |
| --- | --- | --- | --- |
| x | y | xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/and.gify | xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gify |
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | F |

With a conjunction, **both statements must be true for the conjunction to be true**; but with a disjunction, **both statements must be false for the disjunction to be false**. One way to remember this is with the following mnemonic: 'And’ points up to the sand on top of the beach, while ‘or’ points down to the ore deep in the ground.

**Summary:** A **disjunction** is a compound statement formed by joining two statements with the connector OR. The disjunction "p or q" is symbolized by pq. A disjunction is false if and only if both statements are false; otherwise it is true.

**Exercises**

Directions: Read each question below. Select your answer by clicking on its button. Feedback to your answer is provided in the RESULTS BOX. If you make a mistake, choose a different button.

|  |  |
| --- | --- |
| **1.** | **Which of the following sentences is a disjunction?** |
|   | Začátek formulářeAmy played soccer or Bill played hockey.Amy played soccer and Bill played hockey.Amy did not play soccer and Bill played hockey.None of the above. RESULTS BOX: Konec formuláře |

|  |  |
| --- | --- |
| **2.** | **Which of the following statements is a disjunction?** |
|   | Začátek formuláře~xhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/and.gifyxhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/and.gifyxhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifyNone of the above. RESULTS BOX: Konec formuláře |

|  |  |
| --- | --- |
| **3.** | **A disjunction is used with which connector?** |
|   | Začátek formulářeAndOrNotNone of the above. RESULTS BOX: Konec formuláře |

|  |  |
| --- | --- |
| **4.** | **If a is false and b is true, what is the truth value of a**https://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gif~**b?** |
|   | Začátek formulářeTrueFalseNot enough information was givenNone of the above. RESULTS BOX:    Konec formuláře |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **5.** |

|  |  |
| --- | --- |
| **Given:** | **r: y is prime.** |
| **s: y is even.** |
| **Problem:** | **Which of the following is a true statement when y is replaced by 3?** |

 |
|   | Začátek formulářerhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gif~srhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/and.gif~srhttps://www.mathgoodies.com/sites/all/modules/custom/lessons/images/symbolic_logic/images/or.gifsAll of the above. RESULTS BOX:    Konec formuláře |