Solutions

Lesson on Sentences and Negation

1. Which of the following is a closed sentence?

○ Summer follows spring.
○ A quarter is a coin.
○ There are 360 days in a year.
◉ All of the above.

RESULTS BOX:
Correct!

Each of these sentences is a closed sentence (an objective statement which is either true or false).

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2. What is the negation of, "Jenny rides the bus"?

○ Jenny does not like to ride the bus.
◉ Jenny does not ride the bus.
○ Jenny likes to ride the bus.
○ None of the above.

RESULTS BOX:
Correct!

"Jenny does not ride the bus" is the negation of "Jenny rides the bus." The negation of p is "not p."

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3. Which of the following is the negation of x?

○ -x
○ ~(~x)
◉ ~x
○ None of the above.

RESULTS BOX:
Correct!

The statement ~x represents the negation of x.

4.	Given:	a: A triangle is not a polygon.
		b: A square is a rectangle.
	Problem:	Which of the following is the negation of "A triangle is not a polygon"?

- ( ) ~(~b)
- (•) ~a
- ( ) a
- ( ) None of the above.

RESULTS BOX:

Correct!

The statement ~a represents the negation of a.

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5. Which of the following is an open sentence?

- ( ) The number 4 is even.
- ( ) The number 8 is odd.
- ( ) The number 5 is even.
- (•) The number x is odd.

RESULTS BOX:

Correct!

Choice 4 has a variable in it. An open sentence is a statement which contains a variable and becomes either true or false depending on the value that replaces the variable.

Lesson on Conjunction

1. Which of the following sentences is a conjunction?

- ( ) Jill eats pizza or Sam eats pretzels.
- ( ) Jill eats pizza but not pretzels.
- (•) Jill eats pizza and Sam eats pretzels.
- ( ) None of the above.

RESULTS BOX:

Correct!

"Jill eats pizza and Sam eats pretzels" is a conjunction. A conjunction is a compound statement formed by joining two statements with the connector AND.

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2. Which of the following statements is a conjunction?

- ○ p + q
- ◉ p^q
- ○ ~p
- ○ None of the above.

RESULTS BOX:

Correct!

The conjunction "p and q" is symbolized by $p \wedge q$.

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3. A conjunction is used with which connector?

- ○ Not
- ○ Or
- ◉ And
- ○ None of the above.

RESULTS BOX:

Correct!

A conjunction is a compound statement formed by joining two statements with the connector AND.

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4. If a is false and b is true, what is the truth value of $a \wedge b$?

- ○ True
- ◉ False
- ○ Not enough information was given.
- ○ None of the above.

RESULTS BOX:

Correct!

The truth value of $a \wedge b$ is false. A conjunction is true when both of its combined parts are true, otherwise it is false.

5.	Given:	r: y is prime.
		s: y is even.
	Problem:	What is the truth value of $r \wedge s$ when y is replaced by 2?

- (selected) True
- False
- Not enough information was given.
- None of the above.

RESULTS BOX:

Correct!

When y = 2, the statement r is true and the statement s is true (i.e., The number 2 is both prime and even). Therefore, the conjunction $r \wedge s$ is true when y = 2.

Lesson on Disjunction

1. Which of the following sentences is a disjunction?

- (selected) Amy played soccer or Bill played hockey.
- Amy played soccer and Bill played hockey.
- Amy did not play soccer and Bill played hockey.
- None of the above.

RESULTS BOX:

Correct!

A disjunction is a compound statement formed by joining two statements with the connector OR.

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2. Which of the following statements is a disjunction?

- $\sim x \wedge y$
- $x \wedge y$
- (selected) $x \vee y$
- None of the above.

RESULTS BOX:

Correct!

The statement $x \vee y$ is a disjunction.

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3. A disjunction is used with which connector?

- ( ) And
- (•) Or
- ( ) Not
- ( ) None of the above.

RESULTS BOX:

Correct!

A disjunction is a compound statement formed by joining two statements with the connector OR.

---

4. If a is false and b is true, what is the truth value of $a \vee \sim b$?

- ( ) True
- (•) False
- ( ) Not enough information was given
- ( ) None of the above.

RESULTS BOX:

Correct!

If b is true then ~b is false. A disjunction is false when both statements are false. Therefore, the disjunction $a \vee \sim b$ is false.

5.

Given:	**r: y is prime.**
	s: y is even.
Problem:	**Which of the following is a true statement when y is replaced by 3?**

- ( ) $r \vee \sim s$
- ( ) $r \wedge \sim s$
- ( ) $r \vee s$
- (•) All of the above.

RESULTS BOX:

Correct!

When y = 3, the statement r is true and the statement s is false. Therefore, all three choices list true statements.

Lesson on Conditional Statements

1. Which of the following is a conditional statement?

- ◯ Amy plays soccer or Bill plays hockey.
- ◯ Bill plays hockey when Amy plays soccer.
- ◉ If Amy plays soccer then Bill plays hockey.
- ◯ None of the above.

RESULTS BOX:

Correct!

A conditional statement is an if-then statement in which p is a hypothesis and q is a conclusion.

2.

Given:	**r: You give me twenty dollars.**
	s: I will be your best friend.
Problem:	**Which of the following statements represents, "If you give me twenty dollars, then I will be your best friend"?**

- ◯ $r \wedge s$
- ◉ $r \rightarrow s$
- ◯ $s \rightarrow r$
- ◯ None of the above.

RESULTS BOX:

Correct!

The hypothesis is r and the conclusion is s. The logical connector in a conditional statement is denoted by the symbol $\rightarrow$.

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3. **What is the truth value of $r \rightarrow s$ when the hypothesis is false and the conclusion is true in Example 2?**

- (•) True
- ( ) False
- ( ) Not enough information was given.
- ( ) None of the above.

RESULTS BOX:

Correct!

The conditional is defined to be true unless a true hypothesis leads to a false conclusion.

4.

Given:	**a: x is prime.**
	b: x is odd.
Problem:	**What is the truth value of $a \rightarrow b$ when x = 2?**

- ( ) True
- (•) False
- ( ) Not enough information was given.
- ( ) None of the above.

RESULTS BOX:

Correct!

When x=2, hypothesis a is true and conclusion b is false. When a true hypothesis leads to a false conclusion, the conditional is false. Thus when x=2, conditional $a \rightarrow b$ is false.

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5. **What is the truth value of $a \rightarrow b$ when x = 9 in Exercise 4?**

- (•) True
- ( ) False
- ( ) Not enough information was given.
- ( ) None of the above.

RESULTS BOX:

Correct!

When x=9, hypothesis a is false and conclusion b is true. By definition, conditional $a \rightarrow b$ is true.

Lesson on Compound Statements

1.	Given:	a: 11 is prime.
		b: 11 is odd.
	Problem:	Which of the following sentences represents $(a \vee b) \rightarrow \sim b$?

- ○ If 11 is prime and 11 is odd, then 11 is not odd.
- ○ If 11 is prime or 11 is not odd, then 11 is not odd.
- ◉ If 11 is prime or 11 is odd, then 11 is not odd.
- ○ None of the above.

RESULTS BOX:

Correct!

The compound statement $(a \vee b) \rightarrow \sim b$ is a conditional, where the hypothesis is the disjunction "a or b" and the conclusion is $\sim b$.

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2. If r and s are false statements, then what is the truth value of $(\sim r \wedge s) \rightarrow s$?

- ◉ True
- ○ False
- ○ Not enough information was given.
- ○ None of the above.

RESULTS BOX:

Correct!

If r and s are false statements, then $(\sim r \wedge s) \rightarrow s$ is true as shown in the truth table below.

r	s	$\sim r$	$\sim r \wedge s$	$(\sim r \wedge s) \rightarrow s$
F	F	T	F	T

4. What are the truth values of this statement? $(\sim x \vee y) \rightarrow y$

- (selected) {T, T, T, F}
- {T, T, T, T}
- {T, F, T, T}
- None of the above.

RESULTS BOX:

Correct!

The truth values of $(\sim x \vee y) \rightarrow y$ are shown in the truth table below.

x	y	$\sim x$	$\sim x \vee y$	$(\sim x \vee y) \rightarrow y$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	F

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5. What are the truth values of this statement? $\sim p \rightarrow (p \wedge \sim q)$

- {T, F, T, F}
- {F, T, F, T}
- (selected) {T, T, F, F}
- None of the above.

RESULTS BOX:

Correct!

The truth values of $\sim p \rightarrow (p \wedge \sim q)$ are {T, T, F, F},

as shown in the truth table below.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \rightarrow (p \wedge \sim q)$
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	F	F

[Lesson on Biconditional Statements]

1.

Given:	**a: y - 6 = 9**
	b: y = 15
Problem:	**The biconditional a↔b represents which of the following sentences?**

- ○ If y - 6 = 9, then y = 15.
- ◉ y - 6 = 9 if and only if y = 15.
- ○ If y = 15, then y - 6 = 9.
- ○ None of the above.

RESULTS BOX:

Correct!

Biconditional p↔q represents "p if and only if q," where p is a hypothesis and q is a conclusion.

2.

Given:	**r: 11 is prime.**
	s: 11 is odd.
Problem:	**The biconditional r↔s represents which of the following sentences?**

- ○ If 11 is prime, then 11 is odd.
- ○ If 11 is odd, then 11 is prime.
- ◉ 11 is prime iff 11 is odd.
- ○ None of the above.

RESULTS BOX:

Correct!

The hypothesis is "11 is prime" and the conclusion is "11 is odd". So r↔s represents, "11 is prime if and only 11 is odd." The "if and only if" is abbreviated with "iff" in choice 3.

3.	**Given:**	$x \rightarrow y$
		$y \rightarrow x$
	Problem:	**If both of these statements are true then which of the following must also true?**

- ( ) $(x \rightarrow y) \wedge (y \rightarrow x)$
- ( ) $x \leftrightarrow y$
- ( ) x iff y
- (•) All of the above.

RESULTS BOX:

Correct!

When proving the statement p iff q, it is equivalent to proving both of the statements "if p, then q" and "if q, then p". Since these conditionals were given in the problem, $x \leftrightarrow y$ is biconditional. Therefore, each statement listed in choice 1, 2 and 3 is true.

4.	**Given:**	**$m \leftrightarrow n$ is biconditional**
	Problem:	**Which of the following is a true statement?**

- (•) m is the hypothesis
- ( ) m is the conclusion
- ( ) n is a conditional statement
- ( ) n is a biconditional statement

RESULTS BOX:

Correct!

The biconditional $p \leftrightarrow q$ represents "p if and only if q", where p is a hypothesis and q is a conclusion. So m is the hypothesis of $m \leftrightarrow n$.

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5. Which of the following statements is biconditional?

- ( ) I am sleeping if and only if I am snoring.
- ( ) Mary will eat pudding today if and only if it is custard.
- ( ) It is raining if and only if it is cloudy.
- (•) None of the above.

RESULTS BOX:

Correct!

None of these statements is biconditional: one can sleep without snoring; Mary can eat pudding today that is not custard; it can be cloudy without any rain.

Lesson on Tautologies

1. What is the truth value of $r \vee \sim r$?

- (●) True
- ( ) False
- ( ) Not enough information was given.
- ( ) None of the above.

RESULTS BOX:

Correct!

A compound statement, that is always true regardless of the truth value of the individual statements, is defined to be a tautology. The disjunction of a statement and its negation is a tautology.

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2. Is the following statement a tautology? $s \rightarrow \sim s$

- ( ) Yes
- (●) No
- ( ) Not enough information was given.
- ( ) None of the above.

RESULTS BOX:

Correct!

No, the conditional statement $s \rightarrow \sim s$ is not a tautology. See the truth table below.

s	$\sim s$	$s \rightarrow \sim s$
T	F	F
F	T	T

---

3. Is the following statement a tautology? $[(p \vee q) \wedge \sim p] \rightarrow q$

- (selected) Yes
- No
- Not enough information was given.
- None of the above.

RESULTS BOX:

Correct!

Yes, the statement $[(p \vee q) \wedge \sim p] \rightarrow q$

is a tautology since its truth values are {T, T, T, T} as shown in the truth table below.

p	q	~p	$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

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4. Is the following statement a tautology? $\sim(x \vee y) \leftrightarrow (\sim x \wedge \sim y)$

- (selected) Yes
- No
- Not enough information was given.
- None of the above.

RESULTS BOX:

Correct!

Yes, the statement $\sim(x \vee y) \leftrightarrow (\sim x \wedge \sim y)$ is a tautology since its truth values are {T, T, T, T} as shown in the truth table below.

x	y	~x	~y	$x \vee y$	$\sim(x \vee y)$	$\sim x \wedge \sim y$	$\sim(x \vee y) \leftrightarrow (\sim x \wedge \sim y)$
T	T	F	F	T	F	F	T
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	F	T	T	T

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5. Is the following statement a tautology? $a \wedge \sim a$

- ○ Yes
- ◉ No
- ○ Not enough information was given
- ○ None of the above

RESULTS BOX:

Correct!

A conjunction is true when both parts are true. Since a statement and its negation have opposite truth values, the conjunction of a statement and its negation could never be true.

Lesson on Equivalence

1. What are the truth values of the following statement? $(p \wedge \sim q) \rightarrow \sim p$

- ○ {T, T, T, F}
- ◉ {T, F, T, T}
- ○ {F, T, T, T}
- ○ None of the above.

RESULTS BOX:

Correct!

The truth values of $(p \wedge \sim q) \rightarrow \sim p$ are {T, F, T, T} as shown in the truth table below.

p	q	~p	~q	$p \wedge \sim q$	$(p \wedge \sim q) \rightarrow \sim p$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	F	T

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2. Which of the following statements is logically equivalent to the statement given in Exercise 1?

- ○ $q \rightarrow p$
- ○ $\sim p \rightarrow (p \wedge \sim q)$
- ◉ $p \rightarrow q$
- ○ None of the above.

RESULTS BOX:

Correct!

The statement $p \rightarrow q$ is logically equivalent to the statement $(p \wedge \sim q) \rightarrow \sim p$, since they both have the same truth values, as shown in the truth table below.

p	q	$p \rightarrow q$	$(p \wedge \sim q) \rightarrow \sim p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

---

3. Which of the following statements is logically equivalent to $q \rightarrow (p \wedge q)$?

- ◉ $q \rightarrow p$
- ○ $\sim p \rightarrow (p \wedge \sim q)$
- ○ $p \rightarrow q$
- ○ None of the above.

RESULTS BOX:

Correct!

The statement $q \rightarrow p$ is logically equivalent to the statement $q \rightarrow (p \wedge q)$ since they both have the same truth values, as shown in the truth table below.

p	q	$p \wedge q$	$q \rightarrow (p \wedge q)$	$q \rightarrow p$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	F
F	F	F	T	T

---

4. Which of the following statements is logically equivalent to $a \to (a \vee b)$?

- ○ $a \to b$
- ○ $(a \vee b) \to b$
- ◉ $(a \wedge b) \to b$
- ○ None of the above.

RESULTS BOX:

Correct!

The statement $(a \wedge b) \to b$ is logically equivalent to the statement $a \to (a \vee b)$ since they both have the same truth values, as shown in the truth table below.

a	b	$a \vee b$	$a \wedge b$	$(a \wedge b) \to b$	$a \to (a \vee b)$
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	T

5.

Given:	**Statement x is logically equivalent to statement y.**
Problem:	**Which of the following is true?**

- ○ x if and only if y
- ○ $x \leftrightarrow y$ is a tautology
- ○ x iff y
- ◉ All of the above.

RESULTS BOX:

Correct!

Equivalent statements have the same truth values. Therefore, x and y satisfy the definition of a biconditional.. Thus, the statements listed in choice 1 and choice 3 are true. The biconditional of two equivalent statements is a tautology. Therefore, the statement listed in choice 2 is true.

[Practice Exercises]

1.

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	F	T	T
F	F	T	F	F	T

This truth table shows the truth values for the negation of p, and for the conjunction, disjunction and conditional of statements p and q.

2.

p	q	$\sim q$	$p \wedge q$	$(p \wedge q) \rightarrow \sim q$
T	T	F	T	F
T	F	T	F	T
F	T	F	F	T
F	F	T	F	T

This truth table shows the truth values for the compound statement $(p \wedge q) \rightarrow \sim q$.

3.

x	y	$x \rightarrow y$	$y \rightarrow x$	$(x \rightarrow y) \wedge (y \rightarrow x)$	$x \leftrightarrow y$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

This truth table shows the truth values of various compound statements involving x and y.

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4. Which of the following statements from problem 3 is conditional?

- ○ x
- ○ y
- ○ $x \leftrightarrow y$
- ◉ None of the above.

RESULTS BOX:

Correct!

The conditional statements in problem 3 are $x \rightarrow y$ and $y \rightarrow x$.

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5. Which of the following statements from problem 3 is biconditional?

- ○ $x \rightarrow y$
- ○ $y \rightarrow x$
- ◉ $x \leftrightarrow y$
- ○ None of the above.

RESULTS BOX:

Correct!

The biconditional statement from problem 3 is $x \leftrightarrow y$.

6.

a	b	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$[(a \rightarrow b) \wedge a] \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

This truth table shows the truth values of various compound statements involving a and b.

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7. Choose the word that best completes this sentence:
The statement in the last column of the truth table in problem 6 is a ____________.

- ○ Biconditional
- ◉ Tautology
- ○ Disjunction
- ○ None of the above.

RESULTS BOX:

Correct!

The statement in the last column of the truth table in problem 6 is a tautology since all of its truth values are true.

8.

p	q	~q	$p \rightarrow \sim q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \rightarrow \sim q) \leftrightarrow [\sim(p \wedge q)]$
T	T	F	F	T	F	T
T	F	T	T	F	T	T
F	T	T	T	F	T	T
F	F	T	T	F	T	T

The truth values for the last column are all true. Thus the statement $(p \rightarrow \sim q) \leftrightarrow [\sim(p \wedge q)]$ is a tautology.

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9. Which two statements from problem 8 are logically equivalent?

- ○ $p \rightarrow \sim q$ and $p \wedge q$
- ○ $p \wedge q$ and $\sim(p \wedge q)$
- ◉ $p \rightarrow \sim q$ and $\sim(p \wedge q)$
- ○ None of the above.

RESULTS BOX:

Correct!

The statements $p \rightarrow \sim q$ and $\sim(p \wedge q)$ have the same truth value. These statements are, therefore, logically equivalent.

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10. Choose the word that best completes this sentence:
The _____________ of two equivalent statements always yields a tautology.

- (•) Biconditional
- ( ) Conjunction
- ( ) Negation
- ( ) All of the above.

RESULTS BOX:

Correct!

The biconditional of two equivalent statements is a tautology.