**Tautologies**

Example 1: What do you notice about each sentence below?

|  |  |
| --- | --- |
| 1. | A number is even or a number is not even. |
| 2. | Cheryl passes math or Cheryl does not pass math. |
| 3. | It is raining or it is not raining. |
| 4. | A triangle is isosceles or a triangle is not isosceles. |

Each sentence in Example 1 is the disjunction of a statement and its negation. Each of these sentences can be written in symbolic form as por_0.gif~p. Recall that a disjunction is false if and only if both statements are false; otherwise it is true. By this definition, por_0.gif~p is always true, even when statement p is false or statement ~p is false!  This is illustrated in the truth table below:

|  |  |  |
| --- | --- | --- |
| p | ~p | por_0.gif~p |
| T | F | T |
| F | T | T |

The [compound statement](javascript:popUpWindow('compound_statement')) por_0.gif~p consists of the individual statements p and ~p. In the truth table above, por_0.gif~p is always true, regardless of the truth value of the individual statements. Therefore, we conclude that por_0.gif~p is a tautology.

**Definition:** A compound statement, that is always true regardless of the truth value of the individual statements, is defined to be a **tautology**.

Let's look at another example of a tautology.

Example 2: Is (pand_0.gifq)conditional_0.gifp a tautology?

|  |  |  |  |
| --- | --- | --- | --- |
| p | q | pand_0.gifq | (pand_0.gifq)conditional_0.gifp |
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

Solution: The compound statement (pand_0.gifq)conditional_0.gifp consists of the individual statements p, q, and pand_0.gifq. The truth table above shows that (pand_0.gifq)conditional_0.gifp is true regardless of the truth value of the individual statements. Therefore, (pand_0.gifq)conditional_0.gifp is a tautology.

**In the examples below, we will determine whether the given statement is a tautology by creating a truth table.**

Example 3: Is xconditional_0.gif(xor_0.gify) a tautology?

|  |  |  |  |
| --- | --- | --- | --- |
| x | y | xor_0.gify | xconditional_0.gif(xor_0.gify) |
| T | T | T | T |
| T | F | T | T |
| F | T | T | T |
| F | F | F | T |

Solution: Yes; the truth values of xconditional_0.gif(xor_0.gify) are {T, T, T, T}.

Example 4: Is ~bconditional_0.gifb a tautology?

|  |  |  |
| --- | --- | --- |
| b | ~b | ~bconditional_0.gifb |
| T | F | T |
| F | T | F |

Solution: No; the truth values of ~bconditional_0.gifb are {T, F}.

Example 5: Is (por_0.gifq)conditional_0.gif(pand_0.gifq) a tautology?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | (por_0.gifq) | (pand_0.gifq) | (por_0.gifq)conditional_0.gif(pand_0.gifq) |
| T | T | T | T | T |
| T | F | T | F | F |
| F | T | T | F | F |
| F | F | F | F | T |

Solution: No; the truth values of (por_0.gifq)conditional_0.gif(pand_0.gifq) are {T, F, F, T}.

Example 6: Is [(pconditional_0.gifq)and_0.gifp]conditional_0.gifp a tautology?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | pconditional_0.gifq | (pconditional_0.gifq)and_0.gifp | [(pconditional_0.gifq)and_0.gifp]conditional_0.gifp |
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

Solution: Yes; the truth values of [(pconditional_0.gifq)and_0.gifp]conditional_0.gifp are {T, T, T, T}.

Example 7: Is (rconditional_0.gifs)biconditional.gif(sconditional_0.gifr) a tautology?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| r | s | rconditional_0.gifs | sconditional_0.gifr | (rconditional_0.gifs)biconditional.gif(sconditional_0.gifr) |
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

Solution: No; the truth values of (rconditional_0.gifs)biconditional.gif(sconditional_0.gifr) are {T, F, F, T}.

**Summary:** A compound statement that is always true, regardless of the truth value of the individual statements, is defined to be a tautology. We can construct a truth table to determine if a compound statement is a tautology.

**Exercises**

|  |  |
| --- | --- |
| **1.** | **What is the truth value of r**or_0.gif**~r?** |
|  | Začátek formuláře  True False Not enough information was given. None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |
| --- | --- |
| **2.** | **Is the following statement a tautology?  s**conditional_transp.gif**~s** |
|  | Začátek formuláře  Yes No Not enough information was given. None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |
| --- | --- |
| **3.** | **Is the following statement a tautology? [(p**or_0.gif**q)**and_0.gif**~p]**conditional_transp.gif**q** |
|  | Začátek formuláře  Yes No Not enough information was given. None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |
| --- | --- |
| **4.** | **Is the following statement a tautology? ~(x**or_0.gif**y)**biconditional_transp_0.gif**(~x**and_0.gif**~y)** |
|  | Začátek formuláře  Yes No Not enough information was given. None of the above.  RESULTS BOX:  Konec formuláře |

|  |  |
| --- | --- |
| **5.** | **Is the following statement a tautology? a**and_0.gif**~a** |
|  | Začátek formuláře  Yes No Not enough information was given None of the above  RESULTS BOX:  Konec formuláře |